GLOBAL RESEARCH NETWORK
ONLINE RESEARCH HUB

AMERICAN JOURNAL OF SOCIAL AND HUMANITARIAN RESEARCH

ISSN: 2690-9626
Vol.2, No 6, 2021

Construct a Massive Dirac Operator with a Number of Eigenvalues in a Continuous Spectrum

Rashidov Sardor G'ulomovich, Yuldashova Hilola Ataxanovna
Lecturer at Nukus State Pedagogical Institute

ABSTRACT: A massive Dirac operator with a number of eigenvalues $p(x),\ q(x)$ is constructed in the continuous spectrum, and sufficient $L^p(0,\infty)$ conditions are found for this operator to belong to the space of coefficients. The dependence of the eigenvalues $L_2^2(0,\infty)$ of the mass Dirac operator on the continuous spectrum on the general boundary conditions is studied. for the following Dirac operator, which is self-contained in the space of vector functions

$$Dy \equiv B\frac{dy}{dx} + mTy + \Omega(x)y = \lambda y, \quad 0 < x < \infty$$

$\Omega(x) \equiv 0$ in the case of

$$\theta^0(0,\lambda) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \varphi^0(0,\lambda) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

the Weil – Titchmarch function, which satisfies the initial conditions, is defined as a single value.

D The coefficients $p(x),\ q(x)$ of the operator are as follows

$$K(x,y) + F(x,y) + \int_0^x K(x,t)F(t,y)dt = 0, \qquad 0 \le y \le x$$

Found using the Gelfand-Levitan integral equation.

KEYWORD: Dirac operator, operator spectrum, discrete spectrum, continuous spectrum, Weil – Titchmarch function, Gelfand-Levitan integral equation, matrix function, Heveside function.

INTRODUCTION

$L_2^2(0,\infty)$ the following Dirac operator, which is self-contained in the space of vector-functions

$$Dy \equiv B\frac{dy}{dx} + mTy + \Omega(x)y = \lambda\, y, \quad 0 < x < \infty \quad (1)$$

given by the boundary condition

$y_1(0) = 0$, (2)

$$y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \; T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix},$$

m - arbitrary positive number (mass), $p(x), q(x) -$ real continuous functions.

D We divide the spectrum of the operator in the following form

$$\sigma(D) = p\sigma(D) \cup c\sigma(D) \cup pc\sigma(D),$$

where $p\sigma(D)$ - discrete spectrum, $c\sigma(D)$ - continuous spectrum, $pc\sigma(D)$ - set of eigenvalues located in a continuous spectrum.

$\varphi(x,\lambda) = (\varphi_1(x,\lambda), \varphi_2(x,\lambda))^T$ through (1) of the system of equations

$$\varphi_1(0,\lambda) = 0, \qquad \varphi_2(0,\lambda) = 1.$$

we define $f(x) = (\, f_1(x),\; f_2(x)\,)^T$ a solution that satisfies the initial conditions.

$L_2^2(0,\infty)$ Let be a vector-function of any real continuum

$$F(\lambda) = \int_0^\infty [\, f_1(x)\varphi_1(x,\lambda) + f_2(x)\varphi_2(x,\lambda)]dx,$$

The following $f(x)$ Parseval equation in the round depending on the designation

$$\int_0^\infty [\, f_1^{\,2}(x) + f_2^{\,2}(x)]\, dx = \int_{-\infty}^\infty F^2(\lambda)\, d\rho(\lambda). \quad (5)$$

there is a monotonous incremental function called the spectral function of the problem (1), (2) that $\rho(\lambda), \; -\infty < \lambda < \infty$ takes place.

$\Omega(x) \equiv 0$ in this case D we denote by the D_0 operator, i.e.

$$D_0 y \equiv B\frac{dy}{dx} + mTy = \lambda y, \quad 0 < x < \infty \quad (6)$$

$y_1(0) = 0$. (7)

Now of the system of equations (6)

$$\theta^0(0,\lambda)=\begin{pmatrix}1\\0\end{pmatrix},\ \varphi^0(0,\lambda)=\begin{pmatrix}0\\1\end{pmatrix}.$$

we define a solution $\theta^0(x,\lambda)$ that satisfies the initial $\varphi^0(x,\lambda)$ conditions and.

From this

$$\theta^0(x,\lambda)=\begin{pmatrix} cos\left(\sqrt{\lambda^2-m^2}\right)x \\ (\lambda-m)\ \dfrac{sin\left(\sqrt{\lambda^2-m^2}\right)x}{\sqrt{\lambda^2-m^2}} \end{pmatrix},$$

$$\varphi^0(x,\lambda)=\begin{pmatrix} -(\lambda+m)\ \dfrac{sin\left(\sqrt{\lambda^2-m^2}\right)x}{\sqrt{\lambda^2-m^2}} \\ cos\left(\sqrt{\lambda^2-m^2}\right)x \end{pmatrix}.$$

From the following condition

$$\psi^0(x,z)=\theta^0(x,z)+m_0(z)\varphi^0(x,z)\in L_2^2(0,\infty),\ \ Im\,z>0$$

The Weil – Titchmarch function of problems (6) - (7) is defined by a single value

$$m_0(z)=-i\frac{\sqrt{z^2-m^2}}{z+m},\ \ Im\,z>0.$$

In this case, the basis is in the analytical sense, $\sqrt{z^2-m^2}=z+\overset{=}{o}(1),\ \ |z|\to\infty$ and the part that satisfies the condition is obtained.

$m_0(z)$ From the Weil – Titchmarch function and

$$\rho_0(\lambda)-\rho_0(\mu)=\frac{1}{\pi}\lim_{v\to+0}\int_{\mu}^{\lambda}\{-Im\left[m_0(u+iv)\right]\}du$$

using the formula (6) - (7) we find the product of the spectral function of the problem.

$$\rho_0'(\lambda)=\begin{cases} -\dfrac{1}{\pi}\cdot\dfrac{\sqrt{\lambda^2-m^2}}{\lambda+m}, & \lambda<-m, \\ 0, & -m<\lambda<m, \\ \dfrac{1}{\pi}\cdot\dfrac{\sqrt{\lambda^2-m^2}}{\lambda+m}, & \lambda>m. \end{cases}\quad (8)$$

In formula (8), the basis is understood in arithmetic.

(6), (7) The spectrum of problems consists only of a $(-\infty, -m] \cup [m, \infty)$ continuous set, i.e. $c\sigma(D_0) = (-\infty, -m] \cup [m, \infty)$, $p\sigma(D_0) = \varnothing$, $pc\sigma(D_0) = \varnothing$.

Theorem. Let $\{\lambda_n\}_{n=1}^{\infty}$ and $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive numbers that satisfies the following conditions:

1^0. $m < \lambda_1 < \lambda_2 < \ldots < \lambda_n < \ldots$;

2^0. $\sum_{n=1}^{\infty} a_n = c < \infty$;

3^0. $\sum_{n=1}^{\infty} \lambda_n a_n = \tilde{c} < \infty$;

4^0. $\sum_{n=1}^{\infty} a_n \gamma_n = d < 1$,

here

$$\gamma_n = \frac{2}{min\left\{\left|\lambda_{n-1} - \lambda_n\right|, \left|\lambda_{n+1} - \lambda_n\right|\right\}} \cdot \sqrt{\frac{\lambda_1 + m}{\lambda_1 - m}}.$$

It has a continuous and unique matrix function $\Omega(x)$ with the form (3)

$$\rho(\lambda) = \begin{cases} -\dfrac{1}{\pi}\displaystyle\int_{-m}^{\lambda} \frac{\sqrt{t^2 - m^2}}{t+m} dt, & \lambda \le -m, \\ 0 & -m < \lambda \le m, \\ \dfrac{1}{\pi}\displaystyle\int_{m}^{\lambda} \frac{\sqrt{t^2 - m^2}}{t+m} dt + \sum_{n=1}^{\infty} a_n H(\lambda - \lambda_n), & \lambda > m, \end{cases} \quad (9)$$

$H(\lambda)$ (-Hewiside function) is the spectral function of the D operator defined $pc\sigma(D) = \{\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots\}$ by formulas (1) - (3).

Proof. Let us first show that the function defined $\rho(\lambda)$ by formula (9) satisfies all the conditions of the theorem.

$$\rho(\lambda) = \rho_0(\lambda) + \sum_{n=1}^{\infty} a_n H(\lambda - \lambda_n) \quad . (10)$$

is due to the fact that the numbers are positive $\rho_0(\lambda)$ and monotonous. The expression (10) follows

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$$\int_{-\infty}^{\infty} G^2(\lambda)\, d\rho(\lambda) = 0$$

if we put it as an integral

$$\int_{-\infty}^{\infty} G^2(\lambda) d\rho_0(\lambda) + \sum_{n=1}^{\infty} a_n G^2(\lambda_n) = 0.$$

henceforth

$$\int_{-\infty}^{\infty} G^2(\lambda) d\rho_0(\lambda) = 0 \ .$$

If we use the Parseval equation written for problems (6) - (7), then

$$\int_{-\infty}^{\infty} [\ g_1^2(x) + g_2^2(x)]\ dx = 0$$

that is, $g(x)$ from the continuity of the $g(x) \equiv 0$ vector function. So the first condition of Theorem holds.

Now let's look at the following matrix function

$$F(x, y) = \sum_{n=1}^{\infty} a_n \varphi^0(x, \lambda_n) \left(\varphi^0(y, \lambda_n)\right)^T . \ (11)$$

2^0 Conditionally, according to the Weierstrass sign $F(x, y)$ (11), the series is convergent and the function is continuous. $\varphi^0(0, \lambda_n) = 0$ from which $F_{11}(x,0) = 0, \quad F_{21}(x,0) = 0$ equations arise.

Hence, condition II of Theorem also holds. Therefore, the spectral function of the operator, defined by formulas $\rho(\lambda)$ (1) - (3), complements D its continuous spectrum $(-\infty, m]$ and $[m, \infty)$ intervals $\lambda_n, \ n = 1, 2, \ldots$ and has specific values.

D we use the following Gelfand-Levitan integral equation to find the coefficients of the $p(x), \ q(x)$ operator.

$$K(x, y) + F(x, y) + \int_0^x K(x,t) F(t, y) dt = 0, \qquad 0 \le y \le x \ (12)$$

where the nucleus $F(x, y)$ (11) is visible. Each fixed equation has $x \ge 0$ a unique solution. Substituting (11) into (12), then the solution of equation (12) is $K(x, y) = \left\| K_{ij}(x, y) \right\|_{i,\, j=1,2}$ determined as follows

$$K(x,y)=\sum_{n=1}^{\infty}\begin{pmatrix}A_n(x)\\ B_n(x)\end{pmatrix}\cdot\left(\varphi^0(y,\lambda_n)\right)^T \quad . (13)$$

$\begin{pmatrix}A_n(x)\\ B_n(x)\end{pmatrix}$, $n=1,2,\ldots$ To find the vector function, we have the following system of equations.

$$\begin{pmatrix}A_n(x)\\ B_n(x)\end{pmatrix}+a_n\sum_{i=1}^{\infty}\begin{pmatrix}A_i(x)\\ B_i(x)\end{pmatrix}J_{i,n}(x)+a_n\varphi^0(x,\lambda_n)=0, \quad n=1,2,\ldots, (14)$$

here

$$J_{i,n}(x)=\int_0^x\left(\varphi^0(t,\lambda_i)\right)^T\cdot\varphi^0(t,\lambda_n)dt ,$$

that is

$$J_{i,n}(x)=\begin{cases}\dfrac{\varphi_1^0(x,\lambda_i)\varphi_2^0(x,\lambda_n)-\varphi_1^0(x,\lambda_n)\varphi_2^0(x,\lambda_i)}{\lambda_i-\lambda_n}, & i\neq n,\\[2ex] \dfrac{x}{\lambda_n-m}\left(\lambda_n+m\dfrac{\varphi_1^0(x,\lambda_n)\varphi_2^0(x,\lambda_n)}{\lambda_n+m}\right), & i=n.\end{cases}$$

The solution of the system of equations (14) is unique because the system (12) is equivalent to the Gelfand-Levitan integral equation.

If so, $i\neq n$ then

$$\left|J_{i,n}(x)\right|\leq\frac{1}{|\lambda_i-\lambda_n|}\cdot\left\{\sqrt{\frac{\lambda_i+m}{\lambda_i-m}}+\sqrt{\frac{\lambda_n+m}{\lambda_n-m}}\right\}\leq$$

$$\leq\frac{2}{min\left\{\left|\lambda_{n-1}-\lambda_n\right|,\left|\lambda_{n+1}-\lambda_n\right|\right\}}\cdot\sqrt{\frac{\lambda_1+m}{\lambda_1-m}}=\gamma_n. \quad . (15)$$

If so, $i=n$ then

$$J_{n,n}(x)=x+x\frac{m}{\lambda_n-m}\cdot\left\{1-\frac{sin2\left(\sqrt{\lambda_n^2-m^2}\right)x}{2\sqrt{\lambda_n^2-m^2}}\right\}\geq x,$$

$$\frac{1}{a_n}+J_{n,n}(x)\geq\frac{1}{a_n}+x>0,$$

$$\left(\frac{1}{a_n}+J_{n,n}(x)\right)^{-1}\le\left(\frac{1}{a_n}+x\right)^{-1}. \quad .(16)$$

However,

$$\left|\left(J_{i,n}(x)\right)'\right|=\left|\left(\varphi^0(x,\lambda_i)\right)^T\cdot\varphi^0(x,\lambda_n)\right|=$$

$$=\left|(\lambda_i+m)(\lambda_n+m)\frac{\sin\sqrt{\lambda_i^2-m^2}x}{\sqrt{\lambda_i^2-m^2}}\cdot\frac{\sin\sqrt{\lambda_n^2-m^2}x}{\sqrt{\lambda_n^2-m^2}}+\cos\sqrt{\lambda_i^2-m^2}x\cos\sqrt{\lambda_n^2-m^2}x\right|\le$$

$$\le\sqrt{\frac{\lambda_i+m}{\lambda_i-m}}\cdot\sqrt{\frac{\lambda_n+m}{\lambda_n-m}}+1\le M^2+1 \quad ,(17)$$

here. $M=\sqrt{\dfrac{\lambda_1+m}{\lambda_1-m}}$

Then, write the system of equations (14) as follows

$$\begin{pmatrix}A_n(x)\\B_n(x)\end{pmatrix}=-\left(\frac{1}{a_n}+J_{n,n}(x)\right)^{-1}\times\left\{\sum_{\substack{i\ne n\\i=1}}^{\infty}\begin{pmatrix}A_i(x)\\B_i(x)\end{pmatrix}\cdot J_{i,n}(x)+\varphi^0(x,\lambda_n)\right\},\quad n=1,2,\ldots$$

$$\begin{pmatrix}A_n^{(N)}(x)\\B_n^{(N)}(x)\end{pmatrix}=-\left(\frac{1}{a_n}+J_{n,n}(x)\right)^{-1}\times\left\{\sum_{\substack{i\ne n\\i=1}}^{N}\begin{pmatrix}A_i^{(N)}(x)\\B_i^{(N)}(x)\end{pmatrix}\cdot J_{i,n}(x)+\varphi^0(x,\lambda_n)\right\},\quad n=\overline{1,N} \quad ,(18)$$

(19)

that is

$$\sum_{i=1}^{N}\begin{pmatrix}A_i^{(N)}(x)\\B_i^{(N)}(x)\end{pmatrix}\alpha_{i,n}(x)=-\left(\frac{1}{a_n}+J_{n,n}(x)\right)^{-1}\varphi^0(x,\lambda_n),$$

here

$$\alpha_{i,n}(x)=\begin{cases}\left(\dfrac{1}{a_n}+J_{n,n}(x)\right)^{-1}\cdot J_{i,n}(x), & i\ne n,\\ 1, & i=n, \quad i=\overline{1,N}.\end{cases}$$

Since the solution of the integral equation corresponding to system (19) is unique, it follows that the solution of this system of equations is unique.

Let us define the solution of the system of equations (19) as follows.

$$\begin{pmatrix} A_1^{(N)}(x) \\ B_1^{(N)}(x) \end{pmatrix}, \begin{pmatrix} A_2^{(N)}(x) \\ B_2^{(N)}(x) \end{pmatrix}, \dots, \begin{pmatrix} A_N^{(N)}(x) \\ B_N^{(N)}(x) \end{pmatrix}. \quad (20)$$

Now that conditions 1^0 to 4^0 are appropriate, $A_i^{(N)}(x),\ B_i^{(N)}(x),\quad i=\overline{1,N}$ the following estimates are appropriate for solutions

$$\begin{aligned} \left|A_n^{(N)}(x)\right| &\le \frac{a_n M}{1+a_n x}\left\{\frac{c\gamma_n}{1-\alpha}+1\right\}, \\ \left|B_n^{(N)}(x)\right| &\le \frac{a_n}{1+a_n x}\left\{\frac{c\gamma_n}{1-\alpha}+1\right\}. \end{aligned} \quad (21)$$

Let us prove for the expression (21). $A_n^{(N)}(x)$ (19) from the equation

$$A_n^{(N)}(x) = -\left(\frac{1}{a_n}+J_{n,n}(x)\right)^{-1} \times \left\{\sum_{\substack{i\ne n \\ i=1}}^{N} A_i^{(N)}(x) J_{i,n}(x) + \varphi_1^0(x,\lambda_n)\right\}, \quad n=1,2,\dots \quad (22)$$

henceforth

$$\left|A_n^{(N)}(x)\right| \le \left(\frac{1}{a_n}+J_{n,n}(x)\right)^{-1} \times \left\{\sum_{\substack{i\ne n \\ i=1}}^{N} \left|A_i^{(N)}(x)\right| \cdot \left|J_{i,n}(x)\right| + \frac{\lambda_n+m}{\sqrt{\lambda_n^2-m^2}}\right\}.$$

Using evaluations (15) and (16)

$$\left|A_n^{(N)}(x)\right| \le \frac{a_n}{1+a_n x} \cdot \left\{\gamma_n \sum_{i=1}^{N} \left|A_i^{(N)}(x)\right| + M\right\}, \quad n=\overline{1,N}. \quad (23)$$

then

$$\left|A_n^{(N)}(x)\right| \le a_n \cdot \left\{\gamma_n \sum_{i=1}^{N} \left|A_i^{(N)}(x)\right| + M\right\}, \quad n=\overline{1,N}. \quad (24)$$

If we sum up the resulting inequality from 1, then

$$\sum_{n=1}^{\infty} \left|A_n^{(N)}(x)\right| \le \sum_{n=1}^{\infty} a_n\gamma_n \cdot \sum_{i=1}^{N} \left|A_i^{(N)}(x)\right| + M \sum_{n=1}^{\infty} a_n,$$

$$\sum_{n=1}^{\infty} \left|A_n^{(N)}(x)\right|(1-d) \le Mc, \quad \sum_{n=1}^{\infty} \left|A_n^{(N)}(x)\right| \le \frac{Mc}{1-d}.$$

hence from inequality (23)

$$\left| A_n^{(N)}(x) \right| \leq \frac{a_n M}{1 + a_n x} \left\{ \frac{c\gamma_n}{1-d} + 1 \right\}.$$

As a result, inequality (21) was proved.

Now, $\left(A_n^{(N)}(x)\right)'$ we evaluate the product, for which we write (22) as follows:

$$A_n^{(N)}(x) = \sum_{\substack{i \neq n \\ i=1}}^{N} A_i^{(N)}(x) \left\{ -\left(\frac{1}{a_n} + J_{n,n}(x) \right)^{-1} J_{i,n}(x) \right\} + \left\{ -\left(\frac{1}{a_n} + J_{n,n}(x) \right)^{-1} \varphi_1^0(x, \lambda_n) \right\}.$$

Using inequalities (16) and (17

$$\left| \left(A_n^{(N)}(x)\right)' \right| \leq \sum_{\substack{i \neq n \\ i=1}}^{N} \left| \left(A_i^{(N)}(x)\right)' \right| \cdot \frac{a_n \gamma_n}{1 + a_n x} + \sum_{\substack{i \neq n \\ i=1}}^{N} \left| A_i^{(N)}(x) \right| \cdot \left\{ \frac{a_n \gamma_n (1 + M^2)}{(1 + a_n x)^2} + \right.$$

$$\left. + \frac{a_n (1 + M^2)}{1 + a_n x} \right\} + \frac{a_n^2 M (1 + M^2)}{(1 + a_n x)^2} + \frac{(\lambda_n + m) a_n}{1 + a_n x}. \qquad (25)$$

And then

$$\left| \left(A_n^{(N)}(x)\right)' \right| \leq \sum_{i=1}^{N} \left| \left(A_i^{(N)}(x)\right)' \right| \cdot a_n \gamma_n + \sum_{i=1}^{N} \left| A_i^{(N)}(x) \right| \cdot \left\{ a_n^2 \gamma_n (1 + M^2) + a_n (1 + M^2) \right\} +$$

$$+ a_n^2 M (1 + M^2) + (\lambda_n + m) a_n. \qquad (26)$$

If we sum up the resulting inequality from 1, then

$$\sum_{n=1}^{N} \left| \left(A_n^{(N)}(x)\right)' \right| \leq \sum_{n=1}^{N} a_n \gamma_n \cdot \sum_{i=1}^{N} \left| \left(A_i^{(N)}(x)\right)' \right| + (M^2 + 1) \sum_{i=1}^{N} (a_n^2 \gamma_n + a_n) \cdot \sum_{i=1}^{N} \left| A_i^{(N)}(x) \right| +$$

$$+ M (M^2 + 1) \sum_{n=1}^{N} a_n^2 + m \sum_{n=1}^{N} a_n + \sum_{n=1}^{N} \lambda_n a_n.$$

$a_n \gamma_n < d, \quad a_n < c$ using inequalities

$$\sum_{n=1}^{N} \left| \left(A_n^{(N)}(x)\right)' \right| \leq d \sum_{i=1}^{N} \left| \left(A_i^{(N)}(x)\right)' \right| + (1 + d) \cdot (M^2 + 1) c \sum_{i=1}^{N} \left| A_i^{(N)}(x) \right| +$$

$$+ M (M^2 + 1) c^2 + mc + \tilde{c},$$

$$\sum_{n=1}^{N} \left| \left(A_n^{(N)}(x)\right)' \right| \leq \frac{2M (M^2 + 1) c^2 + (1 - d)(mc + \tilde{c})}{(1 - d)^2} = \tilde{d}. \qquad (27)$$

Substituting (27) into (25), then

$$\left|\left(A_n^{(N)}(x)\right)'\right| \le \frac{1}{1+a_n x}\cdot\left\{\tilde{d}\, a_n\gamma_n + \frac{Mc}{1-d}\left[\frac{a_n^2\gamma_n(1+M^2)}{1+a_n x} + (1+M^2)a_n\right] + \right.$$

$$\left. + \frac{a_n^2 M(1+M^2)}{1+a_n x} + (\lambda_n + m)a_n\right\}. \qquad (28)$$

In particular, from (21) and (28) $\left\{A_n^{(N)}(x)\right\}_{N=1}^{\infty}$ and $\left\{\left(A_n^{(N)}(x)\right)'\right\}_{N=1}^{\infty}$ are limited by continuous functions $x \ge 0$ and $n \in N$. According to Arcel's theorem, $[0,\infty)$ there is a partial sequence of n approaching some function for n fixed at $A_n(x)$ each finite $\left\{A_n^{(N_k)}(x)\right\}_{k=1}^{\infty}$ intersection.

$$A_n^{(N_k)}(x) \to A_n(x), \quad k\to\infty, \ \ x\in[0,b],$$

where yeda is a b fixed positive number.

Indeed,

$$B_n^{(N_k)}(x) \to B_n(x), \quad k\to\infty, \ \ x\in[0,b].$$

is displayed in a similar way. $A_n^{(N)}(x)$ and $B_n^{(N)}(x)$ from the continuity of, $A_n(x)$ and $B_n(x)$ will be continuous. From (21)

$$|A_n(x)| \le \frac{a_n M}{1+a_n x}\left\{\frac{c\gamma_n}{1-d}+1\right\}, |B_n(x)| \le \frac{a_n}{1+a_n x}\left\{\frac{c\gamma_n}{1-d}+1\right\}. \ (29)$$

Let us $\left(A_n(x), B_n(x)\right)^T$ now show that a sequence of vector functions satisfies a system of algebraic equations.

Indeed, $\forall\varepsilon>0\ \exists k_0>0$ for

$$\left|A_i(x) - A_i^{(N_k)}(x)\right| < \frac{\varepsilon}{2^i}, \quad k \ge k_0$$

and

$$\left|\sum_{\substack{i\ne n\\ i=1}}^{\infty} A_i(x)\cdot J_{i,n}(x) - \sum_{\substack{i\ne n\\ i=1}}^{N_k} A_i^{(N_k)}(x)\cdot J_{i,n}(x)\right| \le$$

$$\le \sum_{\substack{i\ne n\\ i=1}}^{N_k} \left|A_i(x)\cdot J_{i,n}(x) - A_i^{(N_k)}(x)\cdot J_{i,n}(x)\right|\cdot\left|J_{i,n}(x)\right| +$$

$$+\sum_{\substack{i=N_k+1 \\ i\neq n}}^{\infty} \left|A_i(x)\right| \cdot \left|J_{i,n}(x)\right| \le \sum_{\substack{i\neq n \\ i=1}}^{N_k} \frac{\varepsilon}{2^i}\gamma_n + \varepsilon \ \le \varepsilon \cdot const\,. \qquad (30)$$

From (30)

$$\sum_{\substack{i\neq n \\ i=1}}^{N_k} A_i^{(N_k)}(x) \cdot J_{i,n}(x) \to \sum_{\substack{i\neq n \\ i=1}}^{\infty} A_i(x) \cdot J_{i,n}(x), \quad k \to \infty, \ \ x \in [0,b]\ .$$

If we go to the limit on (19) $N_k \to \infty$ identity, then the sequence (14) $A_n(x), \ \ n = 1,2,\ldots$ satisfies the system.

From the evaluation (29) there will be an approximation of the series (13) according to the Weierstrass sign. Hence, we determine the matrix-function according to the formula (13), $K(x, y)$ in which the coefficients of the operator $p(x)$ and $q(x)$ -s according to the following formulas

$$p(x) = -K_{12}(x,x) - K_{21}(x,x), \qquad q(x) = K_{11}(x,x) - K_{22}(x,x),$$

Or

$$p(x) = -\sum_{n=1}^{\infty}\Big(A_n(x)\varphi_2^0(x,\lambda_n) + B_n(x)\varphi_1^0(x,\lambda_n)\Big),$$

$$q(x) = \sum_{n=1}^{\infty}\Big(A_n(x)\varphi_1^0(x,\lambda_n) + B_n(x)\varphi_2^0(x,\lambda_n)\Big).$$

is defined as a series view.

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