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Natural Numbers and Operations on Them, Fundamentals of the Fundamental Theory of Arithmetic

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ABSTRACT: In this article, you will learn more about natural numbers with respect to their definition, comparison with whole numbers, representation in the number line, properties, etc.

KEYWORD: natural numbers, Closure property, Commutative property, Associative property, Distributive property.

Natural numbers are a part of the number system which includes all the positive integers from 1 till infinity and are also used for counting purpose. It does not include zero (0). In fact, 1,2,3,4,5,6,7,8,9…., are also called counting numbers. Natural numbers are part of real numbers, that include only the positive integers i.e. 1, 2, 3, 4,5,6, ………. Excluding zero, fractions, decimals and negative numbers. Natural numbers do not include negative numbers or zero.

As explained in the introduction part, natural numbers are the numbers which are positive integers and includes numbers from 1 till infinity(∞). These numbers are countable and are generally used for calculation purpose. The set of natural numbers is represented by the letter "N".

N = {1,2,3,4,5,6,7,8,9,10…….}

Natural numbers include all the whole numbers excluding the number 0. In other words, all natural numbers are whole numbers, but all whole numbers are not natural numbers.

Natural Numbers = {1,2,3,4,5,6,7,8,9,…..}

Whole Numbers = {0,1,2,3,4,5,7,8,9,….}

Check out the difference between natural and whole numbers to know more about the differentiating properties of these two sets of numbers. The natural numbers include the positive integers (also known as non-negative integers) and a few examples include 1, 2, 3, 4, 5, 6, …∞. In other words, natural numbers are a set of all the whole numbers excluding 0.

23, 56, 78, 999, 100202, etc. Are all examples of natural numbers.

Natural numbers properties are segregated into four main properties which include:

- Closure property

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- Commutative property
- Associative property
- Distributive property

Each of these properties is explained below in detail.

Natural numbers are always closed under addition and multiplication. The addition and multiplication of two or more natural numbers will always yield a natural number. In the case of subtraction and division, natural numbers do not obey closure property, which means subtracting or dividing two natural numbers might not give a natural number as a result.

- ✓ Addition: 1 + 2 = 3, 3 + 4 = 7, etc. In each of these cases, the resulting number is always a natural number.
- ✓ Multiplication: 2 × 3 = 6, 5 × 4 = 20, etc. In this case also, the resultant is always a natural number.
- ✓ Subtraction: 9 – 5 = 4, 3 – 5 = -2, etc. In this case, the result may or may not be a natural number.
- ✓ Division: 10 ÷ 5 = 2, 10 ÷ 3 = 3.33, etc. In this case, also, the resultant number may or may not be a natural number.

Closure property does not hold, if any of the numbers in case of multiplication and division, is not a natural number. But for addition and subtraction, if the result is a positive number, then only closure property exists.

For example:

-2 x 3 = -6; Not a natural number

6/-2 = -3; Not a natural number

The associative property holds true in case of addition and multiplication of natural numbers i.e. a + (b + c) = (a + b) + c and a × (b × c) = (a × b) × c. On the other hand, for subtraction and division of natural numbers, the associative property does not hold true. An example of this is given below.

- Addition: a + (b + c) = (a + b) + c => 3 + (15 + 1) = 19 and (3 + 15) + 1 = 19.
- Multiplication: a × (b × c) = (a × b) × c => 3 × (15 × 1) = 45 and (3 × 15) × 1 = 45.
- Subtraction: a – (b – c) ≠ (a – b) – c => 2 – (15 – 1) = – 12 and (2 – 15) – 1 = – 14.
- Division: a ÷ (b ÷ c) ≠ (a ÷ b) ÷ c => 2 ÷(3 ÷ 6) = 4 and (2 ÷ 3) ÷ 6 = 0.11.

For commutative property

1. Addition and multiplication of natural numbers show the commutative property. For example, x + y = y + x and a × b = b × a
2. Subtraction and division of natural numbers do not show the commutative property. For example, x – y ≠ y – x and x ÷ y ≠ y ÷ x

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3. Distributive Property: Multiplication of natural numbers is always distributive over addition. For example, $a \times (b + c) = ab + ac$

4. Multiplication of natural numbers is also distributive over subtraction. For example, $a \times (b - c) = ab - ac$

Fundamental Theorem of Arithmetic states that every integer greater than 1 is either a prime number or can be expressed in the form of primes. In other words, all the natural numbers can be expressed in the form of the product of its prime factors. To recall, prime factors are the numbers which are divisible by 1 and itself only. For example, the number 35 can be written in the form of its prime factors as:

$35 = 7 \times 5$

Here, 7 and 5 are the prime factors of 35

Similarly, another number 114560 can be represented as the product of its prime factors by using prime factorization method,

$114560 = 27 \times 5 \times 179$

So, we have factorized 114560 as the product of the power of its primes.

Therefore, every natural number can be expressed in the form of the product of the power of its primes. This statement is known as the Fundamental Theorem of Arithmetic, unique factorization theorem or the unique-prime-factorization theorem.

In number theory, a composite number is expressed in the form of the product of primes and this factorization is unique apart from the order in which the prime factor occurs.

From this theorem we can also see that not only a composite number can be factorized as the product of their primes but also for each composite number the factorization is unique, not taking into consideration order of occurrence of the prime factors.

In simple words, there exists only a single way to represent a natural number by the product of prime factors. This fact can also be stated as:

The prime factorization of any natural number is said to be unique for except the order of their factors.

In general, a composite number "a" can be expressed as,

A = p1 p2 p3 ………… pn, where p1, p2, p3 ………… pn are the prime factors of a written in ascending order i.e. p1≤p2≤p3 ………… ≤pn.

Writing the primes in ascending order makes the factorization unique in nature.

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