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Thermosolutal Effects in Cylindrical Flow with Suction Velocity

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Abstract: *This paper examined the problem of thermosolutal effects in a horizontal cylindrical channel with suction velocity. The fluid flow is assumed to be of constant viscosity, thermal conductivity, diffusivity and it is axi-symmetrical. The problem modelled followed the work of Ize et al.[24] by incorporating suction velocity to their work. Following their method of solution, the governing equations were solved. The analytical results were graphically computed using python programming codes and the graphs showed that fluid velocity increases with increase heat sink, thermal Grashof and solutal Grashof numbers. Also, velocity, energy and species profiles increase with increase in suction velocity while thermal conductivity and diffusivity decrease energy and specie profiles respectively.*

Keywords: *Thermosolutal, heat, solute, suction, velocity, specie.*

1. Introduction

Flow in cylindrical channels with suction velocity is of great relevance in natural processes, science, engineering, medicine, chemical environments to mention a few. These flow types are influenced by coupled heat and mass transport and are applicable in different areas such as oil and gas industries, aerospace, chemical and biological processes, heat exchangers, geothermal power plants and groundwater movements [1]. Due to these applications, several studies on this flow regime have been carried out by so many researchers. For example Isreal-Cookey *et al.*[2], Nwaigwe [3], Deng and Martinez [4], Rundora and Makinde [5], Chinyoka and Makinde[6], Nwaigwe and Makinde [7], Weli and Nwaigwe [8], Bunonyo and Amos[9], Emeka *et al.*[10,11].

The combination of energy and specie transport is referred as Thermosolutal transport. Its effect in cylindrical channels with suction velocity still have great deal of studies due their relevance in chemical, industrial, biological, physical processes and so on. Therefore impact of the thermosolutal transport in porous media have been carried out by some researchers such as Hadidi *et al.*[12] ,Shah *et al.*[13], Corson and Pritchard [14], Mahdy and Ahmed [16], Khan *et al.* [17], Kefayati and Tang[18]. Nwaigwe[19] Futhermore, Nwaigwe [21] numerically solved a heat and mass transport problem in a variable viscous channel flow incorporating solute injection.

In this paper, we extended the work of Ize *et al.*[23] incorporating suction velocity, Frobenius method of power series solution was employed to solve the velocity, energy and specie equations. Our analytical results were graphically displayed to capture the effects of the varying parameters on the governing equations.

This paper is divided into sections as follow: the governing equations including the non-dimensional parameters in section 2, while section 3 displayed the method of solution using power series solution of Frobenius type in details. In section 4 the graphical results of the analytical solutions are displayed to show the effects of the parameters on the governing equations. While the graphical results are discussed in section 5. The paper ends in section 6 with conclusion capturing recommendation of further researches.

2. Mathematical Formulation

Consider the heat and mass transport in a horizontal cylindrical channel. The flow is considered to be in the direction of z^*-axis. The fluid's viscosity and pressure are assumed to be constant. Furthermore, the channel walls are stationary with suction velocity. Therefore, the diagram below assumes r^* to be the distance of the fluid flow from the centre to the channel wall and $\vec{v}^* = (0,0,v^*)$ is the velocity vector. The fluid is also incompressible and Newtonian in nature.

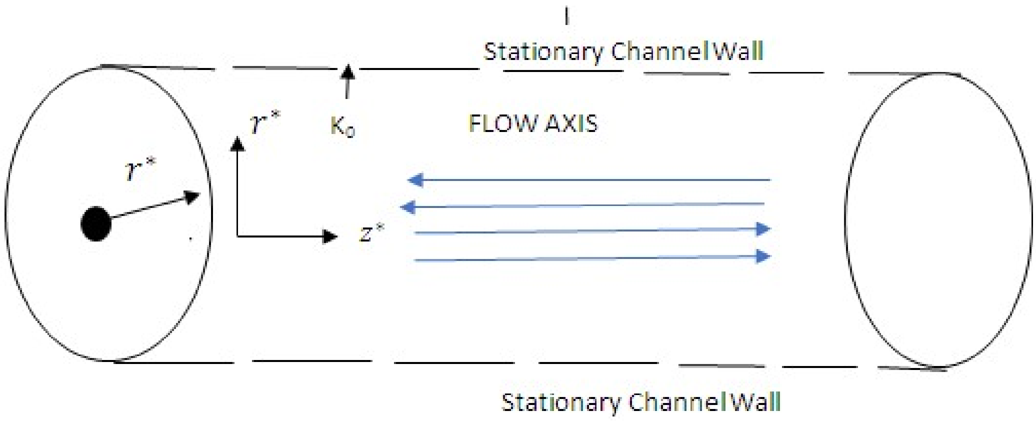

Figure 1: The Physical Model and the Coordinate System of the Fluid Flow.

More conditions imposed are that (i) the fluid flow is of a nonconstant temperature (ii) the flow does varies along z^* and it is also steady (iii) the flow is also axi-symmetric.

Under the assumptions above, the fluid flow is governed by the continuity, energy and specie equations (see the previous of Ize *et al.* [23] for the nondimensional equations) with the following non dimensional quantities:

$$\left.\begin{array}{l} \phi = \dfrac{T^* - T^*_\infty}{T^*_w}, \varphi = \dfrac{C^* - C^*_\infty}{C^*_w}, Gr = \dfrac{g\beta_T T^*_w a^2}{\upsilon_0^2}, Gc = \dfrac{g\beta_C C^*_w a^2}{\upsilon_0^2}, \lambda^2 = \dfrac{Q_0 a^2}{\kappa_0}, \\ z = \dfrac{z^*}{a}, r = \dfrac{r^*}{a}, v = \dfrac{v^*_z a}{\upsilon_0}, P = \dfrac{a^2 P^*}{\rho \upsilon_0^2}, M = B_0 a\sqrt{\dfrac{\sigma_e}{\mu_0}}, C_w = 1 - \dfrac{C^*_\infty}{C^*_w}, \alpha^2 = \dfrac{Q_1 a^2}{D_0} \\ \mu = \dfrac{\mu_f}{\mu_0}, \phi_w = 1 - \dfrac{T^*_\infty}{T^*_w}, \kappa = \dfrac{\kappa^*}{\kappa_0}, k_0 = \dfrac{\rho C p k^*_1 T_w}{\upsilon_0}, D = \dfrac{D_0}{D^*}, p = -\dfrac{\partial P}{\partial z} \end{array}\right\} \quad (1)$$

Equation (1) are introduced into the dimensional equations to give to the following dimensionless equations incorporating suction velocity see Ize *et al.*[23].

$$k_0 \frac{\partial v}{\partial r} = \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right) - M^2 v - p + Gr\phi - Gc\varphi \quad (2)$$

$$k_0 \frac{\partial \phi}{\partial r} = \kappa \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} \right) - \lambda^2 \phi \quad (3)$$

$$k_0 \frac{\partial \varphi}{\partial r} = D \left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} \right) + \alpha^2 \varphi \quad (4)$$

Subject to the boundary conditions

$$\frac{\partial v}{\partial r} = \frac{\partial \phi}{\partial r} = \frac{\partial \varphi}{\partial r} = 0; \quad (5)$$

$$v = 0, \quad \phi_{k_0} = \phi_w, \ \varphi_{k_0} = \varphi_w \ \text{ on } \ r = 1; \quad (6)$$

Where k_0 is the suction velocity, v is fluid velocity component along z^* direction, μ is the viscosity, p is the pressure, M is the magnetic field, Gr is the thermal Grashof number, Gc is the Solutal Grashof number, ϕ is the dimensionless fluid temperature φ is the dimensionless mass concentratio, λ is the heat sink, α is the solute injection, κ is the fluid thermal conductivity, and D is the diffusivity parameter.

Since the fluid flow is dependent on z^* only, equations (2) – (6) become:

$$k_0 \frac{dv}{dr} = \mu \left(\frac{d^2 v}{dr^2} + \frac{1}{r}\frac{dv}{dr} \right) - M^2 v - p + Gr\phi - Gc\varphi \quad (7)$$

$$k_0 \frac{d\phi}{dr} = \kappa \left(\frac{d^2 \phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} \right) - \lambda^2 \phi \quad (8)$$

$$k_0 \frac{d\varphi}{dr} = D \left(\frac{d^2 \varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} \right) + \alpha^2 \varphi \quad (9)$$

Subject to

$$\left. \begin{array}{ll} \dfrac{\partial u}{\partial r} = 0, \dfrac{\partial \theta}{\partial r} = 0, \dfrac{\partial C}{\partial r} = 0 & \text{at } \ r = 0 \\ v = 0, \phi_{k_0} = \phi_w, \varphi_{k_0} = \varphi_w & \text{at } \ r = 1 \end{array} \right\} \quad (10)$$

3. Method of Solution

To solve equations (7)-(10), we assume unique solutions of the forms:

$$v(r) = A_0 v_0(r) + A_1 v_1(r) \quad (11)$$

$$\phi(r) = A_2 \phi_0(r) + A_3 \phi_1(r) \quad (12)$$

$$\varphi(r) = A_4 \varphi_0(r) + A_5 \varphi_1(r) \quad (13)$$

where $v_0(r)$, $v_1(r), \phi_0(r)$, $\phi_1(r)$, $\varphi_0(r)$ and $\varphi_1(r)$ are all linearly independent solutions along with A_0, A_1, A_2, A_3, A_4 and A_5 as arbitrary constants. Therefore, in order to obtain $v_0(r)$, $\phi_0(r)$ and $\varphi_0(r)$, Frobenius power series soluton is adopted as: (see Nwaigwe and Amadi [27] and Ize *et al.*[23]).

$$v_0\left(r\right)=\sum_{s=0}^{\infty} a_s r^{s+m}; \tag{14}$$

$$\phi_0\left(r\right)=\sum_{s=0}^{\infty} \dot{a}_s r^{s+m}; \tag{15}$$

$$\varphi_0\left(r\right)=\sum_{s=0}^{\infty} \ddot{a}_s r^{s+m}; \qquad m=\text{constant} \tag{16}$$

where a_s, $\dot{a}_s$, and $\ddot{a}_s$ are all constants to be obtained.

Therefore, differentiating equations (14) - (16) in their order of degrees in equations (7) – (9) and substituting them into equation (7) – (9) with the boundary conditions (10) applied, the results are obtained:

$$v_0\left(r\right)=r^m\left(\begin{array}{l} a_0 r^0+a_1 r+\left(\dfrac{k_0(m+1)a_1+M^2a_0}{(m+2)^2\mu}\right)r^2+\left(\dfrac{k_0(m+1)a_2+M^2a_1}{(m+3)^2\mu}\right)r^3+\left(\dfrac{k_0(m+1)a_3+M^2a_2}{(m+4)^2\mu}\right)r^4+\left(\dfrac{k_0(m+1)a_4+M^2a_3}{(m+5)^2\mu}\right)r^5 \\ +\left(\dfrac{k_0(m+1)a_5+M^2a_4}{(m+6)^2\mu}\right)r^6+\left(\dfrac{k_0(m+1)a_6+M^2a_5}{(m+7)^2\mu}\right)r^7+\left(\dfrac{k_0(m+1)a_7+M^2a_6}{(m+8)^2\mu}\right)r^8+\left(\dfrac{k_0(m+1)a_8+M^2a_7}{(m+9)^2\mu}\right)r^9 \\ +\left(\dfrac{k_0(m+1)a_9+M^2a_8}{(m+10)^2\mu}\right)r^{10}+\left(\dfrac{k_0(m+1)a_{10}+M^2a_9}{(m+11)^2\mu}\right)r^{11}+\left(\dfrac{k_0(m+1)a_{11}+M^2a_{10}}{(m+12)^2\mu}\right)r^{12}+\ldots \end{array}\right) \tag{17}$$

$$\phi_0\left(r\right)=r^m\left(\begin{array}{l} \dot{a}_0 r^0+\dot{a}_1 r+\left(\dfrac{k_0(m+1)\dot{a}_1+\lambda^2\dot{a}_0}{(m+2)^2\kappa}\right)r^2+\left(\dfrac{k_0(m+1)\dot{a}_2+\lambda^2\dot{a}_1}{(m+3)^2\kappa}\right)r^3+\left(\dfrac{k_0(m+1)\dot{a}_3+\lambda^2\dot{a}_2}{(m+4)^2\kappa}\right)r^4+\left(\dfrac{k_0(m+1)\dot{a}_4+\lambda^2\dot{a}_3}{(m+5)^2\kappa}\right)r^5 \\ +\left(\dfrac{k_0(m+1)\dot{a}_5+\lambda^2\dot{a}_4}{(m+6)^2\kappa}\right)r^6+\left(\dfrac{k_0(m+1)\dot{a}_6+\lambda^2\dot{a}_5}{(m+7)^2\kappa}\right)r^7+\left(\dfrac{k_0(m+1)\dot{a}_7+\lambda^2\dot{a}_6}{(m+8)^2\kappa}\right)r^8+\left(\dfrac{k_0(m+1)\dot{a}_8+\lambda^2\dot{a}_7}{(m+9)^2\kappa}\right)r^9 \\ +\left(\dfrac{k_0(m+1)\dot{a}_9+\lambda^2\dot{a}_8}{(m+10)^2\kappa}\right)r^{10}+\left(\dfrac{k_0(m+1)\dot{a}_{10}+\lambda^2\dot{a}_9}{(m+11)^2\kappa}\right)r^{11}+\left(\dfrac{k_0(m+1)\dot{a}_{11}+\lambda^2\dot{a}_{10}}{(m+12)^2\kappa}\right)r^{12}+\ldots \end{array}\right) \tag{18}$$

$$\varphi_0\left(r\right)=r^m\left(\begin{array}{l} \ddot{a}_0 r^0+\ddot{a}_1 r+\left(\dfrac{k_0(m+1)\ddot{a}_1-\alpha^2\ddot{a}_0}{(m+2)^2 D}\right)r^2+\left(\dfrac{k_0(m+1)\ddot{a}_2-\alpha^2\ddot{a}_1}{(m+3)^2 D}\right)r^3+\left(\dfrac{k_0(m+1)\ddot{a}_3-\alpha^2\ddot{a}_2}{(m+4)^2 D}\right)r^4+\left(\dfrac{k_0(m+1)\ddot{a}_4-\alpha^2\ddot{a}_3}{(m+5)^2 D}\right)r^5 \\ +\left(\dfrac{k_0(m+1)\ddot{a}_5-\alpha^2\ddot{a}_4}{(m+6)^2 D}\right)r^6+\left(\dfrac{k_0(m+1)\ddot{a}_6-\alpha^2\ddot{a}_5}{(m+7)^2 D}\right)r^7+\left(\dfrac{k_0(m+1)\ddot{a}_7-\alpha^2\ddot{a}_6}{(m+8)^2 D}\right)r^8+\left(\dfrac{k_0(m+1)\ddot{a}_8-\alpha^2\ddot{a}_7}{(m+9)^2 D}\right)r^9 \\ +\left(\dfrac{k_0(m+1)\ddot{a}_9-\alpha^2\ddot{a}_8}{(m+10)^2 D}\right)r^{10}+\left(\dfrac{k_0(m+1)\ddot{a}_{10}-\alpha^2\ddot{a}_9}{(m+11)^2 D}\right)r^{11}+\left(\dfrac{k_0(m+1)\ddot{a}_{11}-\alpha^2\ddot{a}_{10}}{(m+12)^2 D}\right)r^{12}+\ldots \end{array}\right) \tag{19}$$

To obtain $v_1(r)$, $\phi_1(r)$ and $\varphi_1(r)$ in equations (14) – (16), equations (17) – (19) are differentiated with respect to m to obtain:

$$v_1(r)=\frac{\partial v_0(r)}{\partial m}=\left[\begin{array}{l} r^m\ln r\left(\begin{array}{l} a_0r^0+a_1r+\left(\frac{k_0(m+1)a_1+M^2a_0}{(m+2)^2\mu}\right)r^2+\left(\frac{k_0(m+1)a_2+M^2a_1}{(m+3)^2\mu}\right)r^3+\left(\frac{k_0(m+1)a_3+M^2a_2}{(m+4)^2\mu}\right)r^4+\left(\frac{k_0(m+1)a_4+M^2a_3}{(m+5)^2\mu}\right)r^5\\ +\left(\frac{k_0(m+1)a_5+M^2a_4}{(m+6)^2\mu}\right)r^6+\left(\frac{k_0(m+1)a_6+M^2a_5}{(m+7)^2\mu}\right)r^7+\left(\frac{k_0(m+1)a_7+M^2a_6}{(m+8)^2\mu}\right)r^8+\left(\frac{k_0(m+1)a_8+M^2a_7}{(m+9)^2\mu}\right)r^9\\ +\left(\frac{k_0(m+1)a_9+M^2a_8}{(m+10)^2\mu}\right)r^{10}+\left(\frac{k_0(m+1)a_{10}+M^2a_9}{(m+11)^2\mu}\right)r^{11}+\left(\frac{k_0(m+1)a_{11}+M^2a_{10}}{(m+12)^2\mu}\right)r^{12}+\ldots \end{array}\right)\\ -r^m\frac{\partial}{\partial m}\left(\begin{array}{l} a_0r^0+a_1r+\left(\frac{k_0(m+1)a_1+M^2a_0}{(m+2)^2\mu}\right)r^2+\left(\frac{k_0(m+1)a_2+M^2a_1}{(m+3)^2\mu}\right)r^3+\left(\frac{k_0(m+1)a_3+M^2a_2}{(m+4)^2\mu}\right)r^4+\left(\frac{k_0(m+1)a_4+M^2a_3}{(m+5)^2\mu}\right)r^5\\ +\left(\frac{k_0(m+1)a_5+M^2a_4}{(m+6)^2\mu}\right)r^6+\left(\frac{k_0(m+1)a_6+M^2a_5}{(m+7)^2\mu}\right)r^7+\left(\frac{k_0(m+1)a_7+M^2a_6}{(m+8)^2\mu}\right)r^8+\left(\frac{k_0(m+1)a_8+M^2a_7}{(m+9)^2\mu}\right)r^9\\ +\left(\frac{k_0(m+1)a_9+M^2a_8}{(m+10)^2\mu}\right)r^{10}+\left(\frac{k_0(m+1)a_{10}+M^2a_9}{(m+11)^2\mu}\right)r^{11}+\left(\frac{k_0(m+1)a_{11}+M^2a_{10}}{(m+12)^2\mu}\right)r^{12}+\ldots \end{array}\right) \end{array}\right] \tag{20}$$

$$\phi_1(r)=\frac{\partial \phi_0(r)}{\partial m}=\left[\begin{array}{l} r^m\ln r\left(\begin{array}{l} \dot{a}_0r^0+\dot{a}_1r+\left(\frac{k_0(m+1)\dot{a}_1+\lambda^2\dot{a}_0}{(m+2)^2\kappa}\right)r^2+\left(\frac{k_0(m+1)\dot{a}_2+\lambda^2\dot{a}_1}{(m+3)^2\kappa}\right)r^3+\left(\frac{k_0(m+1)\dot{a}_3+\lambda^2\dot{a}_2}{(m+4)^2\kappa}\right)r^4+\left(\frac{k_0(m+1)\dot{a}_4+\lambda^2\dot{a}_3}{(m+5)^2\kappa}\right)r^5\\ +\left(\frac{k_0(m+1)\dot{a}_5+\lambda^2\dot{a}_4}{(m+6)^2\kappa}\right)r^6+\left(\frac{k_0(m+1)\dot{a}_6+\lambda^2\dot{a}_5}{(m+7)^2\kappa}\right)r^7+\left(\frac{k_0(m+1)\dot{a}_7+\lambda^2\dot{a}_6}{(m+8)^2\kappa}\right)r^8+\left(\frac{k_0(m+1)\dot{a}_8+\lambda^2\dot{a}_7}{(m+9)^2\kappa}\right)r^9\\ +\left(\frac{k_0(m+1)\dot{a}_9+\lambda^2\dot{a}_8}{(m+10)^2\kappa}\right)r^{10}+\left(\frac{k_0(m+1)\dot{a}_{10}+\lambda^2\dot{a}_9}{(m+11)^2\kappa}\right)r^{11}+\left(\frac{k_0(m+1)\dot{a}_{11}+\lambda^2\dot{a}_{10}}{(m+12)^2\kappa}\right)r^{12}+\ldots \end{array}\right)\\ -r^m\frac{\partial}{\partial m}\left(\begin{array}{l} \dot{a}_0r^0+\dot{a}_1r+\left(\frac{k_0(m+1)\dot{a}_1+\lambda^2\dot{a}_0}{(m+2)^2\kappa}\right)r^2+\left(\frac{k_0(m+1)\dot{a}_2+\lambda^2\dot{a}_1}{(m+3)^2\kappa}\right)r^3+\left(\frac{k_0(m+1)\dot{a}_3+\lambda^2\dot{a}_2}{(m+4)^2\kappa}\right)r^4+\left(\frac{k_0(m+1)\dot{a}_4+\lambda^2\dot{a}_3}{(m+5)^2\bar{\kappa}}\right)r^5\\ +\left(\frac{k_0(m+1)\dot{a}_5+\lambda^2\dot{a}_4}{(m+6)^2\kappa}\right)r^6+\left(\frac{k_0(m+1)\dot{a}_6+\lambda^2\dot{a}_5}{(m+7)^2\kappa}\right)r^7+\left(\frac{k_0(m+1)\dot{a}_7+\lambda^2\dot{a}_6}{(m+8)^2\kappa}\right)r^8+\left(\frac{k_0(m+1)\dot{a}_8+\lambda^2\dot{a}_7}{(m+9)^2\kappa}\right)r^9\\ +\left(\frac{k_0(m+1)\dot{a}_9+\lambda^2\dot{a}_8}{(m+10)^2\kappa}\right)r^{10}+\left(\frac{k_0(m+1)\dot{a}_{10}+\lambda^2\dot{a}_9}{(m+11)^2\kappa}\right)r^{11}+\left(\frac{k_0(m+1)\dot{a}_{11}+\lambda^2\dot{a}_{10}}{(m+12)^2\kappa}\right)r^{12}+\ldots \end{array}\right) \end{array}\right] \tag{21}$$

$$\varphi_1(r)=\frac{\partial\varphi_0(r)}{\partial m}=\left[\begin{array}{l} r^m\ln r\left(\begin{array}{l} a_0r^0+a_1r+\left(\frac{k_0(m+1)a_1-\alpha^2a_0}{(m+2)^2D}\right)r^2+\left(\frac{k_0(m+1)a_2-\alpha^2a_1}{(m+3)^2D}\right)r^3+\left(\frac{k_0(m+1)a_3-\alpha^2a_2}{(m+4)^2D}\right)r^4+\left(\frac{k_0(m+1)a_4-\alpha^2a_3}{(m+5)^2D}\right)r^5\\ +\left(\frac{k_0(m+1)a_5-\alpha^2a_4}{(m+6)^2D}\right)r^6+\left(\frac{k_0(m+1)a_6-\alpha^2a_5}{(m+7)^2D}\right)r^7+\left(\frac{k_0(m+1)a_7-\alpha^2a_6}{(m+8)^2D}\right)r^8+\left(\frac{k_0(m+1)a_8-\alpha^2a_7}{(m+9)^2D}\right)r^9\\ +\left(\frac{k_0(m+1)a_9-\alpha^2a_8}{(m+10)^2D}\right)r^{10}+\left(\frac{k_0(m+1)a_{10}-\alpha^2a_9}{(m+11)^2D}\right)r^{11}+\left(\frac{k_0(m+1)a_{11}-\alpha^2a_{10}}{(m+12)^2D}\right)r^{12}+\ldots \end{array}\right)\\ +r^m\frac{\partial}{\partial m}\left(\begin{array}{l} a_0r^0+a_1r+\left(\frac{k_0(m+1)a_1-\alpha^2a_0}{(m+2)^2D}\right)r^2+\left(\frac{k_0(m+1)a_2-\alpha^2a_1}{(m+3)^2D}\right)r^3+\left(\frac{k_0(m+1)a_3-\alpha^2a_2}{(m+4)^2D}\right)r^4+\left(\frac{k_0(m+1)a_4-\alpha^2a_3}{(m+5)^2D}\right)r^5\\ +\left(\frac{k_0(m+1)a_5-\alpha^2a_4}{(m+6)^2D}\right)r^6+\left(\frac{k_0(m+1)a_6-\alpha^2a_5}{(m+7)^2D}\right)r^7+\left(\frac{k_0(m+1)a_7-\alpha^2a_6}{(m+8)^2D}\right)r^8+\left(\frac{k_0(m+1)a_8-\alpha^2a_7}{(m+9)^2D}\right)r^9\\ +\left(\frac{k_0(m+1)a_9-\alpha^2a_8}{(m+10)^2D}\right)r^{10}+\left(\frac{k_0(m+1)a_{10}-\alpha^2a_9}{(m+11)^2D}\right)r^{11}+\left(\frac{k_0(m+1)a_{11}-\alpha^2a_{10}}{(m+12)^2D}\right)r^{12}+\ldots \end{array}\right) \end{array}\right] \quad (22)$$

Substituting for $m=0$ and writing the equations (20) – (22) in terms of a_0, we obtain the velocity, temperature and concentration solutions as:

$$v(r)=a_0\left(\begin{array}{l} 1+\frac{(Mr)^2}{(2!)^2\mu}+\left(\frac{2k_0M^2r^3}{(3!)^2\mu^2}\right)+\left(\frac{(6M^2k_0^2+9M^4\mu)r^4}{(4!)^2\mu^3}\right)+\left(\frac{(24M^2k_0^2+68M^4k_0\mu)r^5}{(5!)^2\mu^4}\right)\\ +\left(\frac{(120M^2k_0^4+490M^4k_0^2\mu+225M^6\mu^2)r^6}{(6!)^2\mu^5}\right)+\left(\frac{(720M^2k_0^5+3804M^4k_0^3\mu+3798M^6k_0\mu^2)r^7}{(7!)^2\mu^6}\right)\\ +\left(\frac{(5040M^2k_0^6+32508M^4k_0^4\mu+50596M^6k_0^2\mu^2+11025M^8k_0\mu^3)r^8}{(8!)^2\mu^7}\right)\\ +\left(\frac{(40320M^2k_0^7+306144M^4k_0^5\mu+648224M^6k_0^3\mu^2+331272M^8k_0\mu^3)r^9}{(9!)^2\mu^8}\right)+\ldots \end{array}\right)+a_0\left(\ln r\left(v_0(r)\right)+\left.\frac{\partial v_0(r)}{\partial m}\right|_{m=0}\right)$$
$$+\left(\Gamma_0+\Gamma_1r+\Gamma_2r^2+\Gamma_3r^3+\Gamma_4r^4+\Gamma_5r^5+\Gamma_6r^6+\Gamma_7r^7+\Gamma_8r^8+\Gamma_9r^9+\ldots\right) \quad (23)$$

$$\phi(r)=\&_0\left(\begin{aligned}&1+\frac{(\lambda r)^2}{(2!)^2\kappa}+\left(\frac{2k_0\lambda^2r^3}{(3!)^2\kappa^2}\right)+\left(\frac{(6\lambda^2k_0^2+9\lambda^4\kappa)r^4}{(4!)^2\kappa^3}\right)+\left(\frac{(24\lambda^2k_0^2+68\lambda^4k_0\kappa)r^5}{(5!)^2\kappa^4}\right)\\&+\left(\frac{(120\lambda^2k_0^4+490\lambda^4k_0^2\kappa+225\lambda^6\kappa^2)r^6}{(6!)^2\kappa^5}\right)+\left(\frac{(720\lambda^2k_0^5+3804\lambda^4k_0^3\kappa+3798\lambda^6k_0\kappa^2)r^7}{(7!)^2\kappa^6}\right)\\&+\left(\frac{(5040\lambda^2k_0^6+32508\lambda^4k_0^4\kappa+50596\lambda^6k_0^2\kappa^2+11025\lambda^8k_0\kappa^3)r^8}{(8!)^2\kappa^7}\right)\\&+\left(\frac{(40320\lambda^2k_0^7+306144\lambda^4k_0^5\kappa+648224\lambda^6k_0^3\kappa^2+331272\lambda^8k_0\kappa^3)r^9}{(9!)^2\kappa^8}\right)+\ldots\end{aligned}\right)+\&_0\left(\ln r\left(\phi_0(r)\right)+\left.\frac{\partial\phi_0(r)}{\partial m}\right|_{m=0}\right)$$

(24)

$$\varphi(r)=\&_0\left(\begin{aligned}&1-\frac{(\alpha r)^2}{(2!)^2D}-\left(\frac{2k_0\alpha^2r^3}{(3!)^2D^2}\right)+\left(\frac{(-6\alpha^2k_0^2+9\alpha^4D)r^4}{(4!)^2D^3}\right)+\left(\frac{(-24\alpha^2k_0^2+68\alpha^4k_0D)r^5}{(5!)^2D^4}\right)\\&+\left(\frac{(-120\alpha^2k_0^4+490\alpha^4k_0^2D-225\alpha^6D^2)r^6}{(6!)^2D^5}\right)+\left(\frac{(-720\alpha^2k_0^5+3804\alpha^4k_0^3D-3798\alpha^6k_0D^2)r^7}{(7!)^2D^6}\right)\\&+\left(\frac{(-5040\alpha^2k_0^6+32508\alpha^4k_0^4D-50596\alpha^6k_0^2D^2+11025\alpha^8k_0D^3)r^8}{(8!)^2D^7}\right)\\&+\left(\frac{(-40320\alpha^2k_0^7+306144\alpha^4k_0^5D-648224\alpha^6k_0^3D^2+331272\alpha^8k_0D^3)r^9}{(9!)^2D^8}\right)+\ldots\end{aligned}\right)+\&_0\left(\ln r\left(\varphi_0(r)\right)+\left.\frac{\partial\varphi_0(r)}{\partial m}\right|_{m=0}\right)$$

(25)

Hence, the unique solutions for velocity, temperature and concentration as expressed in equations (11) – (13) are obtained as:

$$v(r)=A_0\left(\begin{aligned}&1+\frac{(Mr)^2}{(2!)^2\mu}+\left(\frac{2k_0M^2r^3}{(3!)^2\mu^2}\right)+\left(\frac{(6M^2k_0^2+9M^4\mu)r^4}{(4!)^2\mu^3}\right)+\left(\frac{(24M^2k_0^2+68M^4k_0\mu)r^5}{(5!)^2\mu^4}\right)\\&+\left(\frac{(120M^2k_0^4+490M^4k_0^2\mu+225M^6\mu^2)r^6}{(6!)^2\mu^5}\right)+\left(\frac{(720M^2k_0^5+3804M^4k_0^3\mu+3798M^6k_0\mu^2)r^7}{(7!)^2\mu^6}\right)\\&+\left(\frac{(5040M^2k_0^6+32508M^4k_0^4\mu+50596M^6k_0^2\mu^2+11025M^8k_0\mu^3)r^8}{(8!)^2\mu^7}\right)\\&+\left(\frac{(40320M^2k_0^7+306144M^4k_0^5\mu+648224M^6k_0^3\mu^2+331272M^8k_0\mu^3)r^9}{(9!)^2\mu^8}\right)+\ldots\end{aligned}\right)+A_1\left(\ln r\left(v_0(r)\right)+\left.\frac{\partial v_0(r)}{\partial m}\right|_{m=0}\right)$$
$$+\left(\Gamma_0+\Gamma_1r+\Gamma_2r^2+\Gamma_3r^3+\Gamma_4r^4+\Gamma_5r^5+\Gamma_6r^6+\Gamma_7r^7+\Gamma_8r^8+\Gamma_9r^9+\Gamma_{10}r^{10}\right)$$

(26)

$$\phi(r)=A_2\left(\begin{array}{l}1+\frac{(\lambda r)^2}{(2!)^2\kappa}+\left(\frac{2k_0\lambda^2r^3}{(3!)^2\kappa^2}\right)+\left(\frac{(6\lambda^2k_0^2+9\lambda^4\kappa)r^4}{(4!)^2\kappa^3}\right)+\left(\frac{(24\lambda^2k_0^2+68\lambda^4k_0\kappa)r^5}{(5!)^2\kappa^4}\right)\\+\left(\frac{(120\lambda^2k_0^4+490\lambda^4k_0^2\kappa+225\lambda^6\kappa^2)r^6}{(6!)^2\kappa^5}\right)+\left(\frac{(720\lambda^2k_0^5+3804\lambda^4k_0^3\kappa+3798\lambda^6k_0\kappa^2)r^7}{(7!)^2\kappa^6}\right)\\+\left(\frac{(5040\lambda^2k_0^6+32508\lambda^4k_0^4\kappa+50596\lambda^6k_0^2\kappa^2+11025\lambda^8k_0\kappa^3)r^8}{(8!)^2\kappa^7}\right)\\+\left(\frac{(40320\lambda^2k_0^7+306144\lambda^4k_0^5\kappa+648224\lambda^6k_0^3\kappa^2+331272\lambda^8k_0\kappa^3)r^9}{(9!)^2\kappa^8}\right)+\ldots\end{array}\right)+A_3\left(\ln r\left(\phi_0(r)\right)+\left.\frac{\partial\phi_0(r)}{\partial m}\right|_{m=0}\right) \tag{27}$$

$$\varphi(r)=A_4\left(\begin{array}{l}1-\frac{(\alpha r)^2}{(2!)^2D}-\left(\frac{2k_0\alpha^2r^3}{(3!)^2D^2}\right)+\left(\frac{(-6\alpha^2k_0^2+9\alpha^4D)r^4}{(4!)^2D^3}\right)+\left(\frac{(-24\alpha^2k_0^2+68\alpha^4k_0D)r^5}{(5!)^2D^4}\right)\\+\left(\frac{(-120\alpha^2k_0^4+490\alpha^4k_0^2D-225\alpha^6D^2)r^6}{(6!)^2D^5}\right)+\left(\frac{(-720\alpha^2k_0^5+3804\alpha^4k_0^3D-3798\alpha^6k_0D^2)r^7}{(7!)^2D^6}\right)\\+\left(\frac{(-5040\alpha^2k_0^6+32508\alpha^4k_0^4D-50596\alpha^6k_0^2D^2+11025\alpha^8k_0D^3)r^8}{(8!)^2D^7}\right)\\+\left(\frac{(-40320\alpha^2k_0^7+306144\alpha^4k_0^5D-648224\alpha^6k_0^3D^2+331272\alpha^8k_0D^3)r^9}{(9!)^2D^8}\right)+\ldots\end{array}\right)+A_5\left(\ln r\left(\varphi_0(r)\right)+\left.\frac{\partial\varphi_0(r)}{\partial m}\right|_{m=0}\right) \tag{28}$$

Putting the boundary conditions in equation (10) into equations (26) – (28) and setting $A_1 = A_3 = A_5 = 0$ due to boundedness the following results are expressed as:

$$v(r)=\left(\begin{array}{l}A_0+\Gamma_0r+\left(\frac{M^2A_0}{(2!)^2\mu}+\Gamma_1\right)r^2+\left(\frac{2k_0M^2A_0}{(3!)^2\mu^2}+\Gamma_2\right)r^3+\left(\frac{(6M^2k_0^2+9M^4\mu)A_0}{(4!)^2\mu^3}+\Gamma_3\right)r^4\\+\left(\frac{(24M^2k_0^2+68M^4k_0\mu)A_0}{(5!)^2\mu^4}+\Gamma_4\right)r^5+\left(\frac{(120M^2k_0^4+490M^4k_0^2\mu+225M^6\mu^2)A_0}{(6!)^2\mu^5}+\Gamma_5\right)r^6\\+\left(\frac{(720M^2k_0^5+3804M^4k_0^3\mu+3798M^6k_0\mu^2)A_0}{(7!)^2\mu^6}+\Gamma_6\right)r^7\\+\left(\frac{(5040M^2k_0^6+32508M^4k_0^4\mu+50596M^6k_0^2\mu^2+11025M^8k_0\mu^3)A_0}{(8!)^2\mu^7}+\Gamma_7\right)r^8+\ldots\end{array}\right) \tag{29}$$

$$\phi(r)=\frac{\phi_w}{\phi_{k_0}(1)}\left(\begin{array}{l}1+\frac{(\lambda r)^2}{(2!)^2\kappa}+\left(\frac{2k_0\lambda^2r^3}{(3!)^2\kappa^2}\right)+\left(\frac{(6\lambda^2k_0^2+9\lambda^4\kappa)r^4}{(4!)^2\kappa^3}\right)+\left(\frac{(24\lambda^2k_0^2+68\lambda^4k_0\kappa)r^5}{(5!)^2\kappa^4}\right)\\+\left(\frac{(120\lambda^2k_0^4+490\lambda^4k_0^2\kappa+225\lambda^6\kappa^2)r^6}{(6!)^2\kappa^5}\right)+\left(\frac{(720\lambda^2k_0^5+3804\lambda^4k_0^3\kappa+3798\lambda^6k_0\kappa^2)r^7}{(7!)^2\kappa^6}\right)\\+\left(\frac{(5040\lambda^2k_0^6+32508\lambda^4k_0^4\kappa+50596\lambda^6k_0^2\kappa^2+11025\lambda^8k_0\kappa^3)r^8}{(8!)^2\kappa^7}\right)\\+\left(\frac{(40320\lambda^2k_0^7+306144\lambda^4k_0^5\kappa+648224\lambda^6k_0^3\kappa^2+331272\lambda^8k_0\kappa^3)r^9}{(9!)^2\kappa^8}\right)+\dots\end{array}\right) \tag{30}$$

$$\varphi(r)=\frac{\varphi_w}{\varphi_{k_0}(1)}\left(\begin{array}{l}1-\frac{(\alpha r)^2}{(2!)^2D}-\left(\frac{2k_0\alpha^2r^3}{(3!)^2D^2}\right)+\left(\frac{(-6\alpha^2k_0^2+9\alpha^4D)r^4}{(4!)^2D^3}\right)+\left(\frac{(-24\alpha^2k_0^2+68\alpha^4k_0D)r^5}{(5!)^2D^4}\right)\\+\left(\frac{(-120\alpha^2k_0^4+490\alpha^4k_0^2D-225\alpha^6D^2)r^6}{(6!)^2D^5}\right)+\left(\frac{(-720\alpha^2k_0^5+3804\alpha^4k_0^3D-3798\alpha^6k_0D^2)r^7}{(7!)^2D^6}\right)\\+\left(\frac{(-5040\alpha^2k_0^6+32508\alpha^4k_0^4D-50596\alpha^6k_0^2D^2+11025\alpha^8k_0D^3)r^8}{(8!)^2D^7}\right)+\dots\end{array}\right) \tag{31}$$

where $A_2=\frac{\phi_w}{\phi_{k_0}(1)}$, and $A_4=\frac{\varphi_w}{\varphi_{k_0}(1)}$

4. Results

The graphical results of the analytical solutions for flow velocity, temperature and sepcie solutions are displayed in this section. The three solutions are plotted against different parameters such as viscosity,thermal Grashof, solutal Grashof, magnetic field, thermal conductivity, heat sink, solute injection and diffusivity at varying values, so as to show the effects of suction velocity.

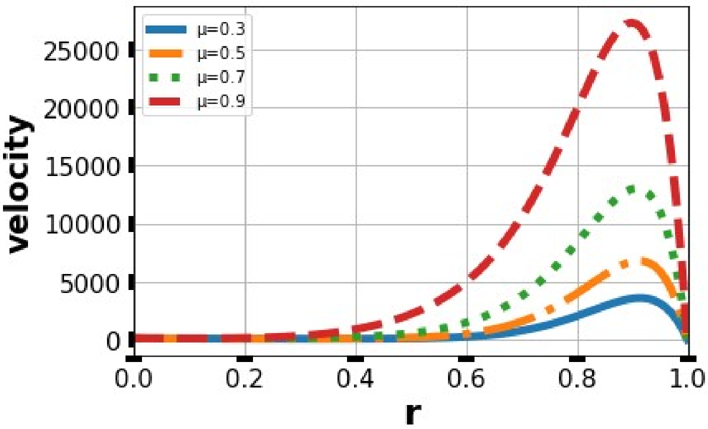

Figure 2: Velocity Profile against r for the values of Gc=0.5, Gr=0.5,α=0.6,κ=0.3, D=5,M=5, k_0 =10, λ=2 varying viscosity parameter.

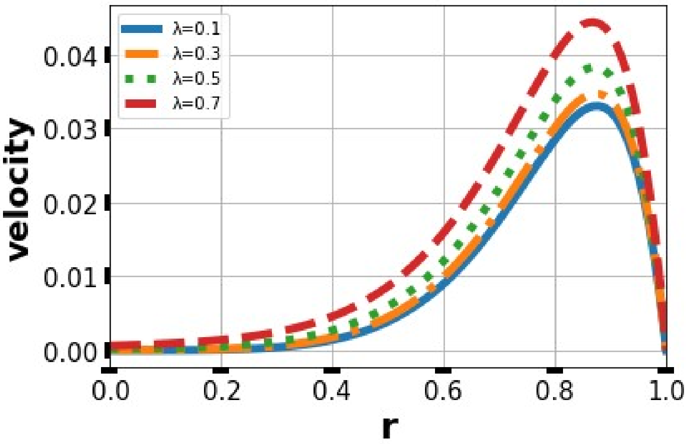

Figure 3: Velocity Profile against r for the values of Gc=0.5, Gr=0.5, α=0.6, κ=0.3,D=5, M=5, k_0=0.3, μ=0.1, p=1.0, varying heat sink parameter

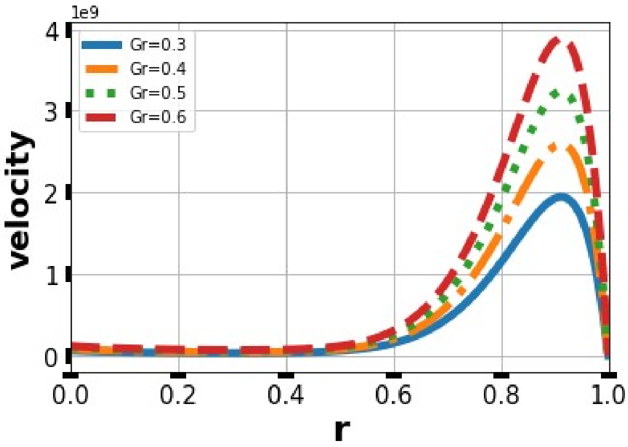

Figure 4: Velocity Profile against r for the values of α=0.6,κ=0.3.D=5,M=5, Gr=0.5, μ=0.1,λ=2, k_0=0.3,p=1.0 varying Thermal Grashof Number

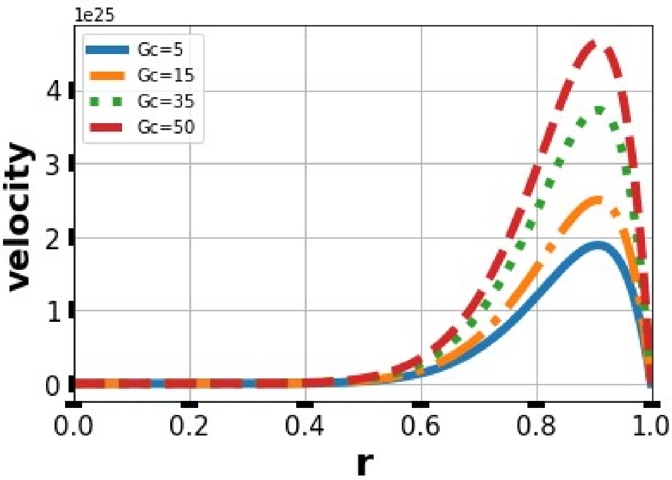

Figure 5: Velocity Profile against r for the values of Gr=15,α=0.6,κ=0.3,D=5,M=0.6, k_0=0.1, μ=0.1, p=1.0, varying Solutal Grashof Number.

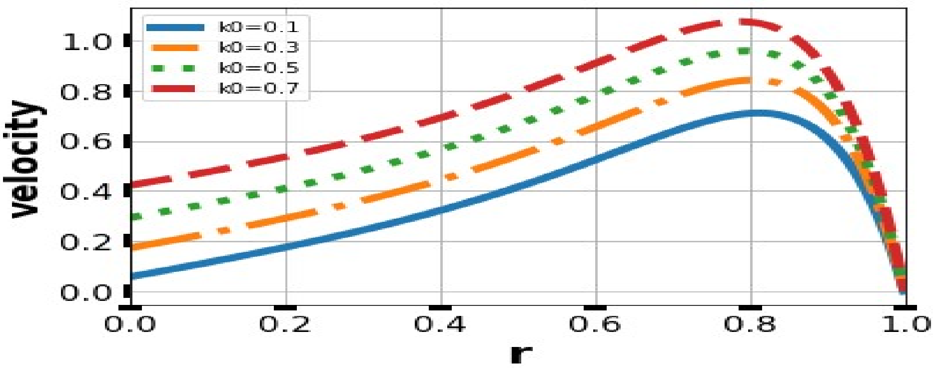

Figure 6: Velocity Profile against r for the values of Gc=0.5, Gr=0.5,κ=0.3,D=5,M=5, μ=0.1, λ=0.8,p=1.0, varying suction parameter.

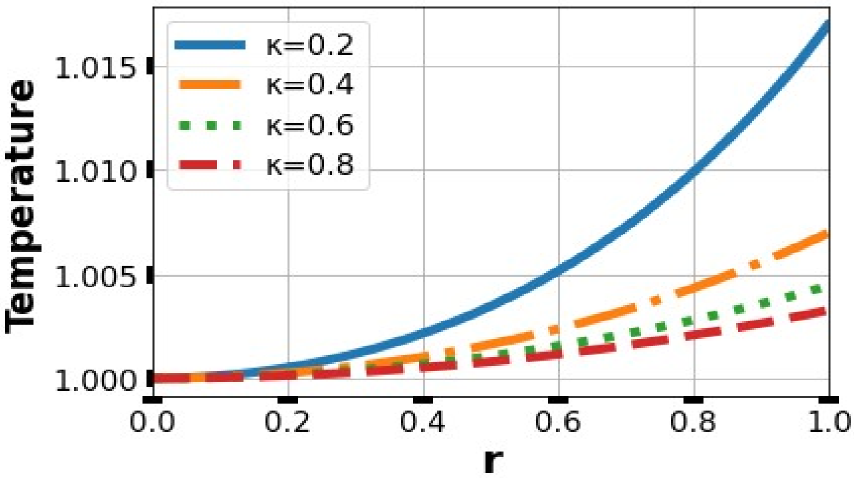

Figure 7: Temperature Profile against r for the values of θw = 0.1, λ = 0.1, k_0= 0.1, varying thermal conductivity parameter.

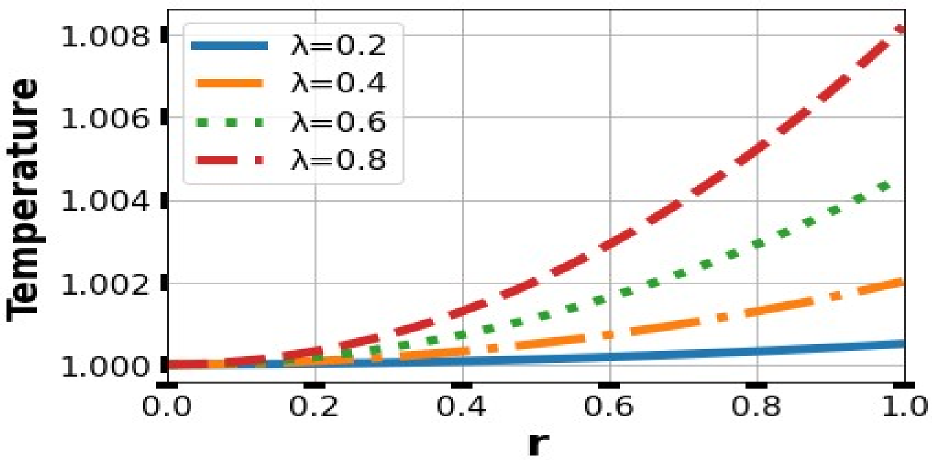

Figure 8: Temperature Profile against r for the values of θw = 0.1, κ = 1.0, k_0= 0.1, varying heat sink parameter.

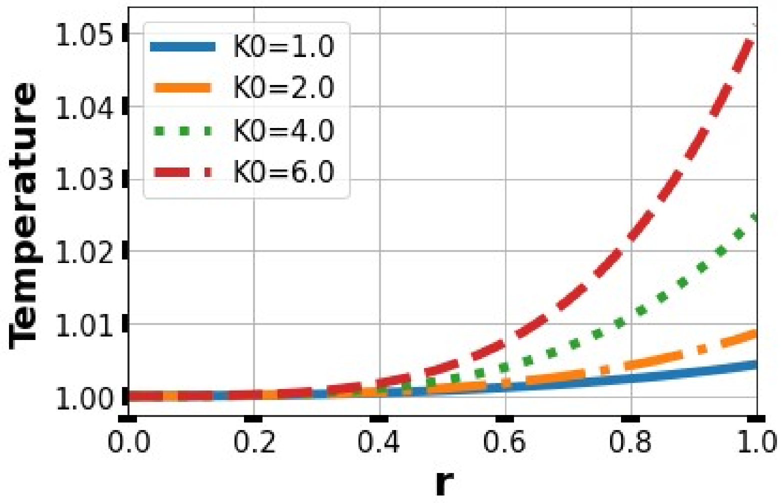

Figure 9: Temperature Profile against r for the values of θw = 0.1, κ = 1.0, λ = 0.1, k_0 = 0.1, varying suction parameter.

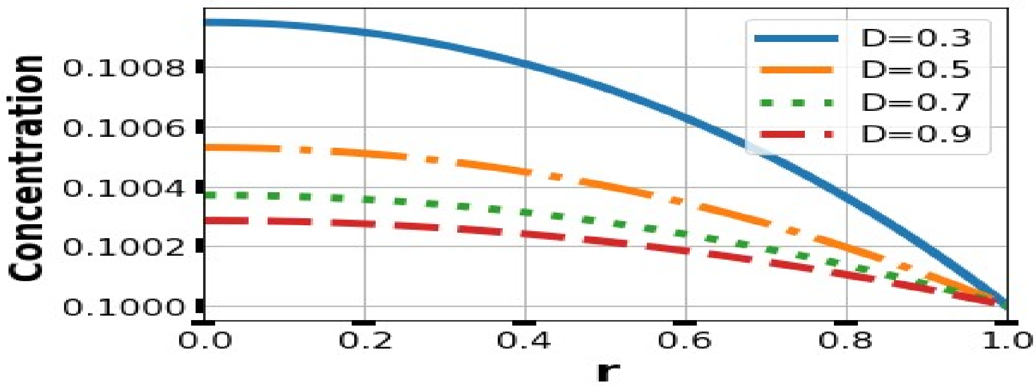

Figure 10: Concentration Profile against r for the values of Cw = 0.1, α = 1.0, k_0=0.1, varying Diffusive parameter.

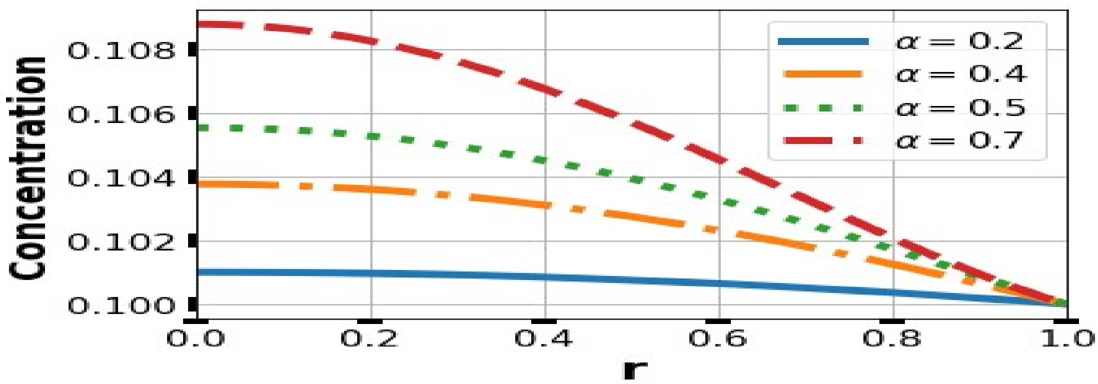

Figure 11: Concentration Profile against r for the values of Cw = 0.1, D= 1.0,K0=0.1, varying solute parameter.

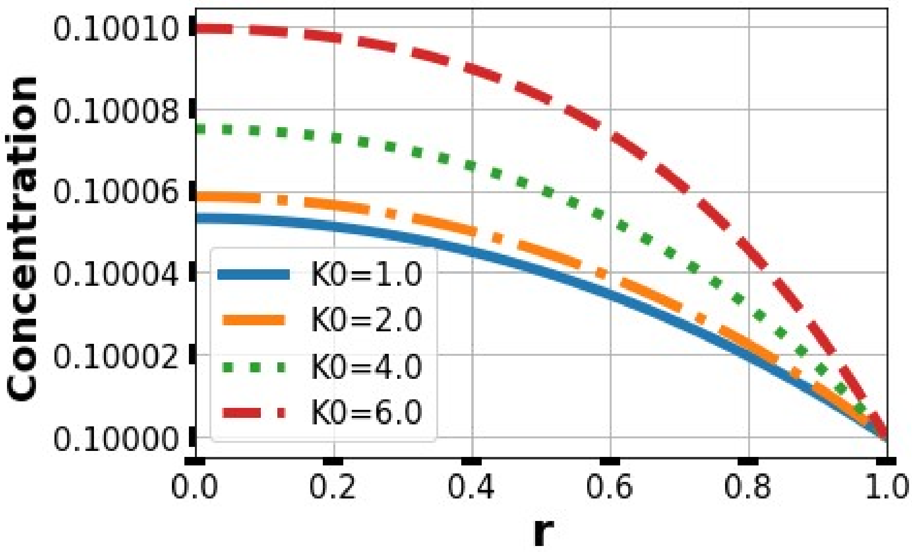

Figure 12: Concentration Profile against r for the values of Cw = 0.1, α = 0.1,D=5.0, varying suction parameter.

5. Discussion

The problem of thermosolutal effects have been solved extending our previous work in Ize *et al*. [23] with the inclusion of suction velocity and it is discovered that, to a large extent, the inclusion has added more value to the work. Therefore, the discussions coming from the study are based on heat sink, solute injection, diffusivity, thermal conductivity, thermal Grashof, Solute Grashof numbers and suction velocity which are our main interest. The discussions are done with the help of graphs and the varied values of the parameters in the problem are indicated in the graphs.

Figure 2 shows the effect of viscosity on the fluid velocity. It is observed that flow velocity increases steadily close to the wall as viscous parameter increases. This is due the fact that suction reduces frictional losses in fluid thereby enhancing fluid velocity see Kishan and Amrutha [15]. In Figure 3, the flow velocity shows no significant change at the centre of the channel as the heat sink is varied but as it moves away from the centre to the wall, velocity increases with heat sink increases. This is in agreement with Asogwa *et al.* [22]

In Figure 4 and 5 the effects of thermal and solutal Grashof numbers on the velocity profile of the flow are shown and observations show that they both increase the fluid velocity as they are varied see Nwaigwe [19].

Figure 6 shows the effect of suction parameter on the fluid velocity distribution. It is discovered that a surge in suction parameter increases the flow velocity and this is more pronounced at the wall where suction reduces the drag of the body of the flow. This is in agreement with Mebine & Adigio [20]. The effect of thermal conductivity parameter on temperature profile is illustrated in Figure 7 and it is discovered that fluid temperature decreases as the thermal conductivity parameter increases. Also Heat sink parameters have effect on fluid temperature in figure 8. It is realised that the fluid temperature increases more at the wall as the cooling parameter increases but no effect is seen at the centre. In Figure 9, the influence of suction on fluid temperature is shown. It is observed that fluid temperature increases at the wall as suction parameter increases.

The significance of diffusive parameter on fluid concentration in figure 10 is displayed. It is discovered that the fluid concentration decreases as the diffusive parameter increases. While Figures 11 illustrates the effect of solute injection parameter on fluid concentration. Observably, increase in the solute injection term depicts increase in the fluid concentration. Also, the suction parameter has an impact on the fluid concentration in figure 12. It is discovered also that, as the suction parameter increases the fluid concentration.

6. Conclusion

In this study, thermosolutal effects in cylindrical channel have been studied and the analytical solutions for velocity, energy and specie equations are displayed graphically using python programming codes. Results showed that fluid velocity increases with increase in parameters such as heat sink, thermal Grashof and solutal Grashof numbers. Also, suction velocity increases the velocity, energy and species profiles while both thermal conductivity and diffusivity decrease energy and specie profiles respectively. Further studies can be done by incorporating variable viscosity and thermal conductivity see Nwaigwe [24]. In this case a stable and convergent numerical method may be needed in both one dimension see Nwaigwe [24,26] and two dimensions [25].

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Appendix

$a_1 = 0$, $a_2 = \dfrac{k_0(m+1)a_1 + M^2 a_0}{(m+2)^2 \mu}$, $a_3 = \dfrac{k_0(m+1)a_2 + M^2 a_1}{(m+3)^2 \mu}$,

$a_4 = \dfrac{k_0(m+1)a_3 + M^2 a_2}{(m+4)^2 \mu}$ $a_5 = \dfrac{k_0(m+1)a_4 + M^2 a_3}{(m+5)^2 \mu}$, $a_6 = \dfrac{k_0(m+1)a_5 + M^2 a_4}{(m+6)^2 \mu}$, $a_7 = \dfrac{k_0(m+1)a_6 + M^2 a_5}{(m+7)^2 \mu}$

$a_8 = \dfrac{k_0(m+1)a_7 + M^2 a_6}{(m+8)^2 \mu}$, $a_9 = \dfrac{k_0(m+1)a_8 + M^2 a_7}{(m+9)^2 \mu}$, $a_{10} = \dfrac{k_0(m+1)a_9 + M^2 a_8}{(m+10)^2 \mu}$

$a_{11} = \dfrac{k_0(m+1)a_{10} + M^2 a_9}{(m+11)^2 \mu}$, $a_{12} = \dfrac{k_0(m+1)a_{11} + M^2 a_{10}}{(m+12)^2 \mu}$, $\dot{a}_1 = 0$, $\dot{a}_2 = \dfrac{k_0(m+1)\dot{a}_1 + \lambda^2 \dot{a}_0}{(m+2)^2 \kappa}$

$\dot{a}_3 = \dfrac{k_0(m+1)\dot{a}_2 + \lambda^2 \dot{a}_1}{(m+3)^2 \kappa}$, $\dot{a}_4 = \dfrac{k_0(m+1)\dot{a}_3 + \lambda^2 \dot{a}_2}{(m+4)^2 \kappa}$, $\dot{a}_5 = \dfrac{k_0(m+1)\dot{a}_4 + \lambda^2 \dot{a}_3}{(m+5)^2 \kappa}$ $\dot{a}_6 = \dfrac{k_0(m+1)\dot{a}_5 + \lambda^2 \dot{a}_4}{(m+6)^2 \kappa}$,

$\dot{a}_7 = \dfrac{k_0(m+1)\dot{a}_6 + \lambda^2 \dot{a}_5}{(m+7)^2 \kappa}$, $\dot{a}_8 = \dfrac{k_0(m+1)\dot{a}_7 + \lambda^2 \dot{a}_6}{(m+8)^2 \kappa}$

$\dot{a}_9 = \dfrac{k_0(m+1)\dot{a}_8 + \lambda^2 \dot{a}_7}{(m+9)^2 \kappa}$, $\dot{a}_{10} = \dfrac{k_0(m+1)\dot{a}_9 + \lambda^2 \dot{a}_8}{(m+10)^2 \kappa}$, $\dot{a}_{11} = \dfrac{k_0(m+1)\dot{a}_{10} + \lambda^2 \dot{a}_9}{(m+11)^2 \kappa}$

$\dot{a}_{12} = \dfrac{k_0(m+1)\dot{a}_{11} + \lambda^2 \dot{a}_{10}}{(m+12)^2 \kappa}$, $\ddot{a}_1 = 0$, $\ddot{a}_2 = \dfrac{k_0(m+1)\ddot{a}_1 - \alpha^2 \ddot{a}_0}{(m+2)^2 D}$, $\ddot{a}_3 = \dfrac{k_0(m+1)\ddot{a}_2 - \alpha^2 \ddot{a}_1}{(m+3)^2 D}$

$\ddot{a}_4 = \dfrac{k_0(m+1)\ddot{a}_3 - \alpha^2 \ddot{a}_2}{(m+4)^2 D}$, $\ddot{a}_5 = \dfrac{k_0(m+1)\ddot{a}_4 - \alpha^2 \ddot{a}_3}{(m+5)^2 D}$, $\ddot{a}_6 = \dfrac{k_0(m+1)\ddot{a}_5 - \alpha^2 \ddot{a}_4}{(m+6)^2 D}$

$\ddot{a}_7 = \dfrac{k_0(m+1)\ddot{a}_6 - \alpha^2 \ddot{a}_5}{(m+7)^2 D}$, $\ddot{a}_8 = \dfrac{k_0(m+1)\ddot{a}_7 - \alpha^2 \ddot{a}_6}{(m+8)^2 D}$, $\ddot{a}_9 = \dfrac{k_0(m+1)\ddot{a}_8 - \alpha^2 \ddot{a}_7}{(m+9)^2 D}$

$\ddot{a}_{10} = \dfrac{k_0(m+1)\ddot{a}_9 - \alpha^2 \ddot{a}_8}{(m+10)^2 D}$, $\ddot{a}_{11} = \dfrac{k_0(m+1)\ddot{a}_{10} - \alpha^2 \ddot{a}_9}{(m+11)^2 D}$, $\ddot{a}_{12} = \dfrac{k_0(m+1)\ddot{a}_{11} - \alpha^2 \ddot{a}_{10}}{(m+12)^2 D}$

$$A_0 = \frac{-\left(\Gamma_0+\Gamma_1+\Gamma_2+\Gamma_3+\Gamma_4+\Gamma_5+\Gamma_6+\Gamma_7+\Gamma_8+\Gamma_9+\Gamma_{10}\right)}{\begin{array}{l} 1+\frac{M^2}{(2!)^2\mu}+\left(\frac{2k_0M^2}{(3!)^2\mu^2}\right)+\left(\frac{6M^2k_0^2+9M^4\mu}{(4!)^2\mu^3}\right)+\left(\frac{24M^2k_0^2+68M^4k_0\mu}{(5!)^2\mu^4}\right) \\ +\left(\frac{120M^2k_0^4+490M^4k_0^2\mu+225M^6\mu^2}{(6!)^2\mu^5}\right)+\left(\frac{720M^2k_0^5+3804M^4k_0^3\mu+3798M^6k_0\mu^2}{(7!)^2\mu^6}\right) \\ +\left(\frac{5040M^2k_0^6+32508M^4k_0^4\mu+50596M^6k_0^2\mu^2+11025M^8k_0\mu^3}{(8!)^2\mu^7}\right) \\ +\left(\frac{40320M^2k_0^7+306144M^4k_0^5\mu+648224M^6k_0^3\mu^2+331272M^8k_0\mu^3}{(9!)^2\mu^8}\right)+\ldots \end{array}}$$

$$\Gamma_{11}=\frac{Gr\beta_{11}-Gc\chi_{11}}{M^2},\qquad \Gamma_{10}=\frac{Gr\beta_{10}-Gc\chi_{10}-11k_0\Gamma_{11}}{M^2},\qquad \Gamma_9=\frac{Gr\beta_9-Gc\chi_9-10k_0\Gamma_{10}+132\mu\Gamma_{11}}{M^2}$$

$$\Gamma_8=\frac{Gr\beta_8-Gc\chi_8-9k_0\Gamma_9+100\mu\Gamma_{10}}{M^2},\qquad \Gamma_7=\frac{Gr\beta_7-Gc\chi_7-8k_0\Gamma_8+81\mu\Gamma_9}{M^2},$$

$$\Gamma_6=\frac{Gr\beta_6-Gc\chi_6-7k_0\Gamma_7+64\mu\Gamma_8}{M^2},$$

$$\Gamma_5=\frac{Gr\beta_5-Gc\chi_5-6k_0\Gamma_6+49\mu\Gamma_7}{M^2}\quad \Gamma_4=\frac{Gr\beta_4-Gc\chi_4-5k_0\Gamma_5+36\mu\Gamma_6}{M^2},$$

$$\Gamma_3=\frac{Gr\beta_3-Gc\chi_3-4k_0\Gamma_4+25\mu\Gamma_5}{M^2},$$

$$\Gamma_2=\frac{Gr\beta_2-Gc\chi_2-3k_0\Gamma_3+16\mu\Gamma_4}{M^2},$$

$$\Gamma_1=\frac{Gr\beta_1-Gc\chi_1-2k_0\Gamma_2+9\mu\Gamma_3}{M^2}\quad \Gamma_0=\frac{-p+Gr\beta_0-Gc\chi_0-k_0\Gamma_1+4\mu\Gamma_2}{M^2},\ \beta_0=\frac{\phi_w}{\phi_0(1)}.\qquad \beta_1=\frac{\lambda^2}{(2!)^2\kappa}.\beta_0,$$

$$\beta_2=\frac{2\lambda^2k_0}{(3!)^2\kappa^2}.\beta_0\qquad \beta_3=\frac{(6\lambda^2k_0^2+9\lambda^4\kappa)}{(4!)^2\kappa^3}.\beta_0,\qquad \beta_4=\frac{(24\lambda^2k_0^3+68\lambda^4k_0\kappa)}{(5!)^2\kappa^4}.\beta_0,$$

$$\beta_5=\frac{(120\lambda^2k_0^4+490\lambda^4k_0^2\kappa+225\lambda^6\kappa^2)}{(6!)^2\kappa^5}.\beta_0,\quad \beta_6=\frac{(720\lambda^2k_0^5+3804\lambda^4k_0^3\kappa+3798\lambda^6k_0\kappa^2)}{(7!)^2\kappa^6}.\beta_0$$

$$\beta_7=\frac{(5040\lambda^2k_0^6+32508\lambda^4k_0^4\kappa+50596\lambda^6k_0^2\kappa^2+11025\lambda^8\kappa^3)}{(8!)^2\kappa^7}.\beta_0,$$

$$\beta_8=\frac{(40320\lambda^2k_0^7+306144\lambda^4k_0^5\kappa+648224\lambda^6k_0^3\kappa^2+31272\lambda^8k_0\kappa^3)}{(9!)^2\kappa^8}.\beta_0,$$

$$\beta_9=\frac{(362880\lambda^2k_0^8+683769\lambda^4k_0^6\kappa+8467164\lambda^6k_0^4\kappa^2+7079724\lambda^8k_0^2\kappa^3+893025\lambda^{10}\kappa^4)}{(10!)^2\kappa^9}.\beta_0,$$

$$\beta_{10}=\frac{(3628800\lambda^2k_0^9+10869690\lambda^4k_0^7\kappa+115286040\lambda^6k_0^5\kappa^2+135619640\lambda^8k_0^3\kappa^3+42057450\lambda^{10}\kappa^4)}{(11!)^2\kappa^{10}}.\beta_0$$

$$\chi_0=\frac{\varphi_w}{\varphi_0(1)},\quad \chi_1=\frac{-\alpha^2}{(2!)^2D}.\chi_0,\quad \chi_2=\frac{-2\alpha^2k_0}{(3!)^2D^2}.\chi_0,$$

$$\chi_3=\frac{(-6\alpha^2k_0^2+9\alpha^4D)}{(4!)^2D^3}.\chi_0,\ \chi_4=\frac{(-24\alpha^2k_0^2+68\alpha^4k_0D)}{(5!)^2D^4}.\chi_0,\ \chi_5=\frac{(-120\alpha^2k_0^4+490\alpha^4k_0^2D-225\alpha^6D^2)}{(6!)^2D^5}.\chi_0,$$

$$\chi_6=\frac{(-720\alpha^2k_0^5+3804\alpha^4k_0^3D-3798\alpha^6k_0D^2)}{(7!)^2D^6}.\chi_0,$$

$$\chi_7=\frac{(-5040\alpha^2k_0^6+32508\alpha^4k_0^4D-50596\alpha^6k_0^6D^2+11025\alpha^8D^3)}{(8!)^2D^7}.\chi_0,$$

$$\chi_7=\frac{(-5040\alpha^2k_0^6+32508\alpha^4k_0^4D-50596\alpha^6k_0^6D^2+11025\alpha^8D^3)}{(8!)^2D^7}.\chi_0,$$

$$\chi_8=\frac{(-40320\alpha^2k_0^7+306144\alpha^4k_0^5D-648224\alpha^6k_0^3D^2+331272\alpha^8D^3)}{(9!)^2D^8}.\chi_0,$$

$$\chi_9=\frac{(-362880\alpha^2k_0^8+683769\alpha^4k_0^6D-8467164\alpha^6k_0^4D^2+7079724\alpha^8k_0^2D^3-893025\alpha^{10}D^4)}{(10!)^2D^9}.\chi_0,$$

$$\chi_{10}=\frac{(-3628800\alpha^2k_0^9+10869690\alpha^4k_0^7D-115286040\alpha^6k_0^5D^2+135619640\alpha^8k_0^3D^3-42057450\alpha^{10}k_0D^4)}{(11!)^2D^{10}}.\chi_0$$

$$\phi_{k_0}(1)=1+\frac{\lambda^2}{(2!)^2\kappa}+\frac{2k_0\lambda^2}{(3!)^2\kappa^2}+\frac{(6\lambda^2k_0^2+9\lambda^4\kappa)}{(4!)^2\kappa^3}+\frac{(24\lambda^2k_0^2+68\lambda^4k_0\kappa)}{(5!)^2\kappa^4}$$
$$+\frac{(120\lambda^2k_0^4+490\lambda^4k_0^2\kappa+225\lambda^6\kappa^2)}{(6!)^2\kappa^5}+\frac{(720\lambda^2k_0^5+3804\lambda^4k_0^3\kappa+3798\lambda^6k_0\kappa^2)}{(7!)^2\kappa^6}$$
$$+\frac{(5040\lambda^2k_0^6+32508\lambda^4k_0^4\kappa+50596\lambda^6k_0^2\kappa^2+11025\lambda^8k_0\kappa^3)}{(8!)^2\kappa^7}$$
$$+\frac{(40320\lambda^2k_0^7+306144\lambda^4k_0^5\kappa+648224\lambda^6k_0^3\kappa^2+331272\lambda^8k_0\kappa^3)}{(9!)^2\kappa^8}+\ldots$$

$$\varphi_{k_0}(1)=1-\frac{\alpha^2}{(2!)^2D}-\frac{2k_0\alpha^2}{(3!)^2D^2}+\frac{(-6\alpha^2k_0^2+9\alpha^4D)}{(4!)^2D^3}+\frac{(-24\alpha^2k_0^2+68\alpha^4k_0D)}{(5!)^2D^4}$$
$$+\frac{(-120\alpha^2k_0^4+490\alpha^4k_0^2D-225\alpha^6D^2)}{(6!)^2D^5}+\frac{(-720\alpha^2k_0^5+3804\alpha^4k_0^3D-3798\alpha^6k_0D^2)}{(7!)^2D^6}$$
$$+\frac{(-5040\alpha^2k_0^6+32508\alpha^4k_0^4D-50596\alpha^6k_0^2D^2+11025\alpha^8k_0D^3)}{(8!)^2D^7}+\ldots$$

