

## Statistical Analysis of Compressive Strength of Concrete

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### ABSTRACT

Concrete is used in the construction practice as it is easily available, cheap, has flexibility of handling and can be shaped in any form desired. The use of concrete in the construction industry is now being done for a very long time. It is the most used material in the parts of structure where compressive strength is required. So in the construction process evaluation of the compressive strength of concrete plays a vital role. There is no universal method for the evaluation of the compressive strength. In India, the method prescribed by IS 456:2000 is used for checking whether the concrete made is fulfilling the requirement. An attempt is made to analyse this method statistically. The characteristic strength of the concrete is represented by test results of concrete cubes at 28 days. The compressive strength of concrete in India is defined based on this characteristic compressive strength. The sampling process of the concrete which is based on the amount of work of concrete to be done is studied and a trend between the number of samples and the failure percentage of samples in terms of compressive strength is studied. The concept of population and sample in terms of the concrete mix design is studied. Hypothesis testing is performed on various groups of samples and their deviation from the population is calculated. An attempt to calculate optimal number of samples for testing and an improvised method using statistics as a tool for checking the compressive strength of concrete is made.

## 1. INTRODUCTION

The concrete as a material is a mixture of aggregates, coarse and fine mixed with cement paste which becomes hard after a period of time. It is a composite material. It is one of the most commonly used man made material in the construction industry all over the world. Its usage worldwide in the construction industry is about twice that of steel, wood, aluminium and plastic combined. One of the reasons for wide use of concrete in the construction is because of its sustainability. Concrete is a natural choice for sustainable construction as it is environment friendly than other construction materials throughout its life span, starting from raw material production up to demolition. The following are some of the factors given by the Environmental Council of Concrete Organizations and the Portland Cement Association that shows the sustainability of concrete [1].

“The principal raw material in the manufacturing of Portland cement is limestone, and it is a mineral that exists in huge quantities naturally. As a substitute of cement, fly ash, silica fumes, slag, etc. is also being used widely. All the waste by products from the steel mills, power plants,

and other manufacturing facilities canal so be putouts. The structures built from concrete material are long lasting and durable which will not burn rot or rust very easily. The life span of the concrete structures is much longer than the other common building structures. Concrete overlays on pavement prevent water entering into the ground surface because of their can be reduced because of the water-resistive characteristics. This tends to create problems such as water table depletion, erosion, flash floods and pollution which creates imbalance in natural ecosystem Pervious concrete is a unique concrete possessing inter-connected pores within it that allows free flow of solutions. The structures like parking lots, drive ways, pavements, sidewalks, etc. use this type of concrete that helps in the detainment and restoration of water. The above factors shows some of the reasons why the concrete is extensively used in, bridges, buildings, dams and roads with its usage ranging from structural applications pipes, kerbs, drains, etc. The concrete used in these structures can also be of varying types. The following are few of the types of commonly used concrete that are used.

### Objectives

- A detailed study of various statistical methods applicable to concrete material.
- Statistical analysis of the compressive strength of 100cubes of M30 grade of concrete.
- Determining the optimal number of cubes required to check whether the required amount of strength of concrete is achieved or not.
- Performing analysis for obtaining an equation for determining the compressive strength of the concrete of an obtained mixture.

## 2. LITERATUREREVIEW

Analysis of variance or ANOVA is a statistical method which has statistical models and various estimation procedures associated with it and is used for analyzing the difference among group means in a sample. This method was developed by Ronald Fisher who is a renowned statistician. The law of total variance is the basis for this method, where a given variable's observed variance is broken down into different components, each of which is then assigned to a different source of variation. In its most basic forms, the ANOVA is a statistical test that determines whether or not the means of two or more populations are equal. When it comes to actual testing, it tells us weather a particular property of concrete is being affected by variability in mix proportions, exposure conditions, curing conditions, etc. The individual effect of the variable as well as a combined effect of two or more variables on the property of concrete canals be determined.

In one of the research conducted, the use of ANOVA was done to see the effect of varying proportion of silica fumes and polypropylene fibres along with the variation in curing temperature on both concrete's flexural and compressive strengths. The percentage contribution of variation for each property to the compressive and flexural strengths was determined using ANOVA [16].

“It is a confirmatory data analysis method which is used to check whether or not the assumed hypothesis is true or false. In a hypothesis test, two statements one known as null hypothesis and other as alternate hypothesis about the population are evaluated and the statement which supports the sample data is determined. To compare the accuracy of the data obtained through hypothesis, we either need two different data sets to compare the mutually owned a data set obtained from synthetic set, which is verified against the data set calculated through hypothesis. The synthetic data set is actually based over the idealized model. The testing is done by comparing the critical and calculated p-value of the statistical data according to the given distribution. Confidence intervals can be expressed in a way by the hypothesis tests which are based on statistical significance So in away, every confidence interval can be obtained by doing significance based hypothesis testing and based on significance every hypothesis test can be obtained via a

confidence interval. For the frame work of statistical hypothesis testing, significance based hypothesis testing is most commonly used [18].

The use of hypothesis testing can be widely found in literatures present. To give an example, a study on the effects of the variation of natural aggregates and recycle aggregate on compressive strength was done. To determine whether the test results of recycled aggregate concrete, regular aggregate concrete, and mixtures containing various proportions of natural and recycled aggregates follow a normal distribution, hypothesis testing was conducted. It was assumed that the distribution is normal, which is the null hypothesis. Since the value determined through statistics is lesser than that given by critical value in hypothesis, therefore the null hypothesis undertaken to determine the value statistically is acceptable [19].

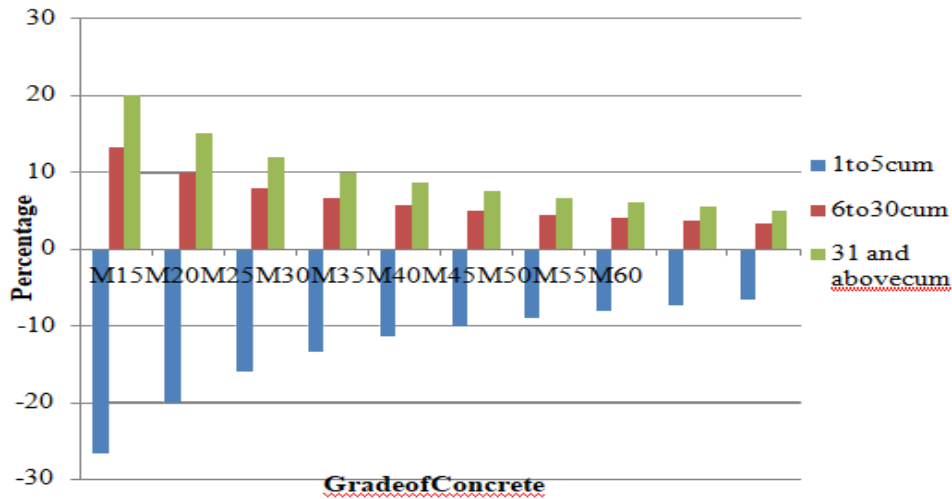
The normality test in statistics is used to check if the obtained data set is modelled properly by normal distribution and to compute whether it is likely for a data set underlying random variable to be normally distributed. The K-S test or Kolmogorov–Smirnov test is a commonly used hypothesis test used to find whether the given set of data follows normal distribution. The p-value for the normal distribution is calculated and if the obtained value is less than the significance level or the critical p-value the data is said to follow the normal distribution [22].

### 3. COMPRESSIVE STRENGTH FOR CRITICAL VALUES

The values of the samples and the group of samples below which the test results are not expected to fall are known as the critical values of compressive strength. These figures are determined using the codal rules specified in IS 456:2000. "In accordance with IS 456 for the scenario where the value of standard deviation is taken from the table of assumed standard deviations, the values given in tables 1 and 2, along with the criteria that the value of compressive strength of individual specimens should be within 15% of the average of the specimens which is the sample value, is sufficient to check the compressive strength of concrete mixture. The values in the table 1 and table 2 are subjected to change if standard deviation other than the assumed standard deviation is established by actually casting 30 samples. The table 1 shows the minimum value of individual test result of compressive strength (in MPa) that can be obtained according to the grade of concrete for varying volume of quantity of concrete work as per the codal provision of IS 456:2000." [10][23]

**Table 1. Minimum value of test results of each sample of concrete**

Grade of Concrete	Values of strength for varying volume of concrete (MPa)		
	1 m <sup>3</sup> to 5 m <sup>3</sup>	6 m <sup>3</sup> to 30 m <sup>3</sup>	31 m <sup>3</sup> and above
M15	19	13	12
M20	24	18	17
M25	29	23	22
M30	34	28	27
M35	39	33	32
M40	44	38	37
M45	49	43	42
M50	54	48	47
M55	59	53	52
M60	64	58	57

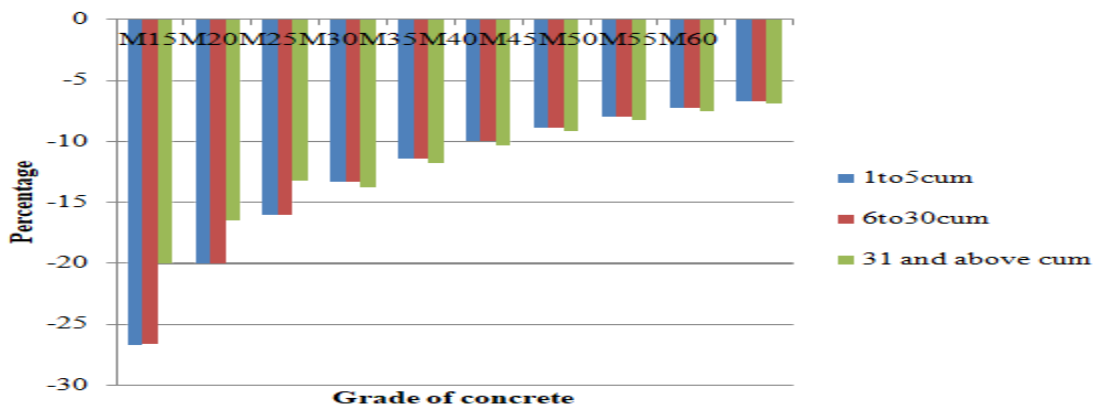


**Figure 1. Grade of concrete vs percentage difference between f<sub>ck</sub> and minimum value of test result**

The figure 1 shows the percentage difference between the characteristic compressive strength (f<sub>ck</sub>) of concrete and the minimum value of individual test result of compressive strength for varying volume of quantity of concrete work. The table 2 shows the minimum value of mean of group of all the samples that can be obtained according to the grade of concrete as per the codal provision of IS456:2000.

**Table 2. Minimum value of mean of group of all sample of concrete (inMPa)**

Grade of Concrete	Values of strength for varying volume of concrete		
	1 m <sup>3</sup> to 5m <sup>3</sup>	6 m <sup>3</sup> to 30m <sup>3</sup>	31 m <sup>3</sup> and above
M15	19	19	18
M20	24	24	23.3
M25	29	29	28.3
M30	34	34	34.1
M35	39	39	34.1
M40	44	44	44.1
M45	49	49	49.1
M50	54	54	54.1
M55	59	59	59.1
M60	64	64	64.1



**Figure 2. Grade of concrete vs percentage difference between f<sub>ck</sub> and mean of all samples**

The figure 2 shows the percentage difference between the concrete's characteristic compressive strength (fck) and the lowest mean value of the group of all samples for different amounts of concrete work. Thus, even though the test results show that the compressive strength obtained is higher than the characteristic compressive strength, it is evident that in the majority of cases, the concrete mixture is rejected. The assumed standard deviation's value is basically the basis for the calculation of these values. There is no proper basis for the values of assumed standard deviations as well as the equations of the acceptance criteria. A relation between the entire concrete mixture and the casted samples needs to be established. This can be done by the analysis of population and samples.

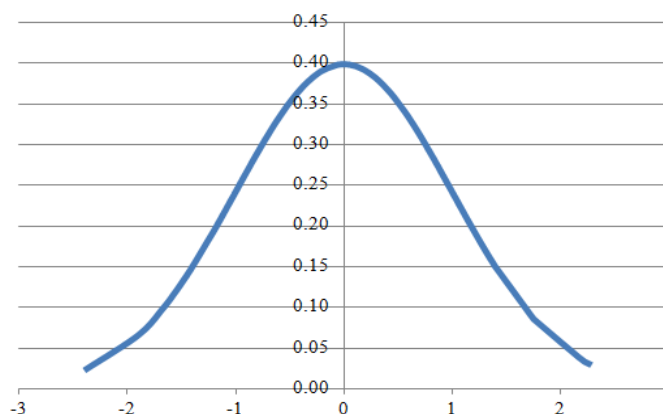
### ANALYSIS OF POPULATION AND SAMPLE

Statistical population is the entire set of similar items like in case of compressive strength of concrete, the strength of the entire mixture can be termed as population. "A sample is a subset of population which represents the entire population. Sampling is the method which is used in the estimation of the population characteristics from a subset taken from the same population. An error in sampling can lead to misleading conclusions regarding the populations, so it is important that the sampling is done at a proper qualitative and quantitative basis." [12] Hypothesis testing is a method in statistical analysis where a relation between sample and population can be established. So for the analysis, considering M30 grade of concrete a random data of 100 numbers between 27 and 45 was generated using MATLAB coding software, details of which are given in table 3. These 100 values which were rounded off to the second decimal point were considered as the compressive strength of 100 cubes and the data was further evaluated.

**Table 3. Details of compressive strength values of 100 cubes**

<b>Characteristic Compressive Strength(fck)</b>	30 MPa
<b>Mean(favg)</b>	35.82MPa
<b>Standard Deviation(<math>\sigma</math>)</b>	3.63MPa

These values were then plotted to see whether they follow a normal distribution. The figure 1 shows the plot of 100 values, and as we are getting a smooth bell shaped curve, it can be assumed that the values follow normal distribution.



**Figure 3. Distribution of assumed random data**

The cubes are then divided in specimens of 75,50,30,25,20,18,16,15,14,13,12,11,10 and 9. Hypothesis testing is then performed on each group of specimens to check whether they belong to the population. 50000000 such different combinations from each group were tested for 99% confidence interval. The table 4 shows the percentage of total specimens tested. The numbers of combinations from each group that passed and failed the hypothesis test were calculated and are

shown in the table 5. The interval for the number of specimen in each sample is decreased as the rate at which the number of samples that are failing is increasing.

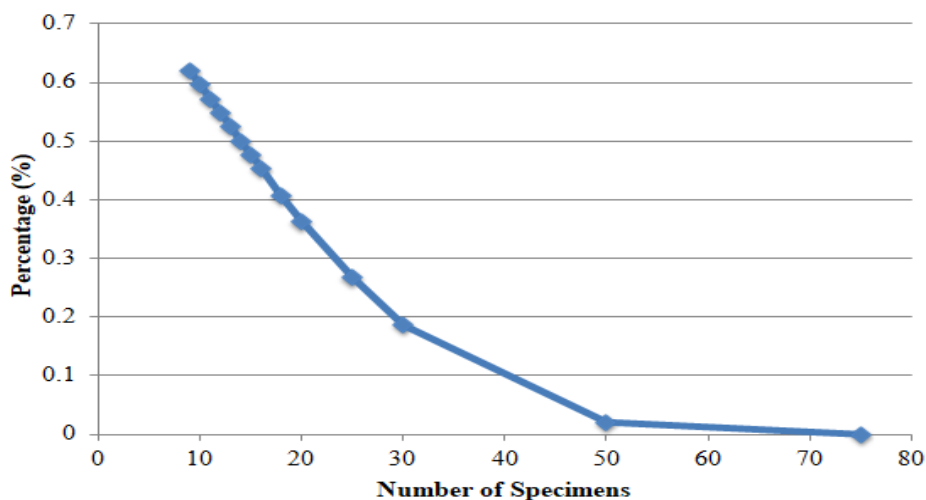
**Table 4. Percentage of the tested combinations from the total number of combinations**

Number of Specimens	Percentage of total combinations taken
75	$2.06 \times 10^{-14}$
50	$4.96 \times 10^{-20}$
30	$1.70 \times 10^{-16}$
25	$2.06 \times 10^{-14}$
20	$9.33 \times 10^{-12}$
18	$1.63 \times 10^{-10}$
16	$3.72 \times 10^{-9}$
15	$1.97 \times 10^{-8}$
14	$1.13 \times 10^{-7}$
13	$7.03 \times 10^{-7}$
12	$4.76 \times 10^{-6}$
11	$3.53 \times 10^{-5}$
10	$2.89 \times 10^{-4}$
9	$2.63 \times 10^{-3}$

**Table 5. Results of Hypothesis Testing**

Number of Specimens in each sample	Number of Samples Passing the Hypothesis	Number of Samples Failing the Hypothesis
75	50000000	0
50	49989632	10368
30	49906940	93060
25	49866036	133964
20	49818189	181811
18	49796987	203013
16	49772951	227046
15	49761279	238721
14	49750593	249407
13	49737452	262548
12	49725967	274033
11	49713861	286139
10	49702150	297850
9	49690287	309713

The numbers of combinations from each group that passed and failed the hypothesis test were calculated and are shown in the table5. The interval for the number of specimen in each sample is decreased as the rate at which the number of samples that are failing is increasing. The graph showing of percentage failure of each group which is the percentage of combinations failing the hypothesis test over the total combinations taken for each group is plotted as shown in the figure 3. The graph shows that as the number of specimens is decreasing the percentage of failure increases.



**Figure 4. Percentage Failure vs Number of Specimens**

## CONCLUSIONS

The conclusions that could be derived from the work done are as follows: -

- The calculation of compressive strength of concrete of the given grade as per Indian Codal Provisions given in IS 456:2000 is largely based on the table of assumed standard deviations and equations which do not have a strong basis.
- In some cases even the samples in which values obtained more than the required characteristic compressive strength are rejected.
- As the number of specimens in the sample decrease, the number of samples passing the hypothesis of being from the population decreases

The work can be useful in the following ways

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