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## Ikki O'lchovli Muhitda Moddaning Anomal Ko'Chishi

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Ba'zan tezlik va dispersiya uchun ifodalar degenerativ shaklda yoziladi. Ikki o'lchovli holatda suspenziyaning ko'chishi ham bo'ylama, ham ko'ndalang yo'nalishda sodir bo'ladi. Ko'ndalang yo'nalish bo'ylab sezilarli darajada suspenziyalarni ko'chishi, hatto ularning bo'ylama yo'nalishga nisbatan juda past ko'ndalang tezlik va dispersiyada ham qayd etiladi. Bu shuni ko'rsatadiki, 2D model 1D modelga qaraganda ko'proq mos keladi [2].

Bu yerdada ikki o'lchovli ob'ekt ko'rib chiqiladi. Muhitning ma'lum bir nuqtasidan ma'lum bir konsentratsiyali eritma yuboriladi. Bunday nuqta manbadan eritma o'zaro perpendikulyar yo'nalishlarda  $x$  muhitga tarqaladi ( $0 \leq x < \infty$ ;  $0 \leq y < \infty$ ) va  $y$  hududning  $(x, y)$  ma'lum bir nuqtasida oqim tezligining komponentlari  $x$  va  $y$  yo'nalishlari mos ravishda  $u(x, t)$  va  $v(y, t)$  bilan belgilang. Bu ikkala komponent Darsi qonunini qanoatlantiradi.  $D_x(x, t)$  va  $D_y(y, t)$  gidrodinamik dispersiyaning bo'ylama va ko'ndalang komponentlari mos ravishda  $x$  va  $y$  yo'nalishlarda [1].

Ikki o'lchovli holatda konvektiv diffuziya tenglamasini quyidagi ko'rinishda qaraymiz:

$$\begin{aligned} \frac{\partial C(x, y, t)}{\partial t} = & \frac{\partial}{\partial x} \left( D_x(x, t) \frac{\partial C(x, y, t)}{\partial x} - u(x, t) C(x, y, t) \right) + \\ & + \frac{\partial}{\partial y} \left( D_y(y, t) \frac{\partial C(x, y, t)}{\partial y} - v(y, t) C(x, y, t) \right), \end{aligned} \quad (1)$$

Bu yerda  $C(x, y, t)$  - modda konsentratsiyasi.

Ikki o'lchovli adveksiya-dispersiya (1) tenglamasini yechish uchun boshlang'ich va chegara shartlarini qo'yish kerak.

Dastlab, muhit toza (moddasiz) suyuqlik bilan to'ldirilgan. Dastlabki vaqt momentidan boshlab ma'lum bir konsentratsiyaga ega bo'lgan birjinslimas suyuqlik  $(0, 0)$  nuqtadan beriladi. Keyin boshlang'ich va chegaraviy shartlarni quyidagicha yozish mumkin [2]

$$C(x, y, t) = 0, \quad x \geq 0; \quad y \geq 0, \quad t = 0, \quad (2)$$

$$C(x, y, t) = \begin{cases} C_0, & x = 0; \quad y = 0; \quad 0 < t \leq t_0 \\ 0, & x = 0; \quad y = 0; \quad t > t_0 \end{cases} \quad (3)$$

$$\frac{\partial C(x, y, t)}{\partial x} = 0, \quad x \rightarrow \infty; \quad \frac{\partial C(x, y, t)}{\partial y} = 0, \quad y \rightarrow \infty; \quad t \geq 0, \quad (4)$$

Muhit bir hil bo'lmaganligi sababli, ikkita tezlik komponenti, ya'ni  $u(x, t)$  va  $v(y, t)$ , mos keladigan  $x$  va  $y$  koordinatalarning chiziqli funksiyalari deb hisoblanadi. Bundan tashqari, tezliklar  $t$  ga bog'liq deb hisoblanadi, ya'ni tezlik komponentining ba'zi funksional bog'liqligi  $t$  hisobga olinadi. Shunday qilib, suyuqlik harakati tezligining tarkibiy qismlari shaklda olinadi

$$u(x, t) = u_0 f_1(mt)(1 + ax), \quad v(y, t) = v_0 f_1(mt)(1 + by), \quad (5)$$

Bu yerda  $a$  va  $b$  - bo'ylama va ko'ndalang yo'nalishlarda bir xillik parametrlari. Turli ma'nolar  $a$  va  $b$  bir jinslilikning turli xususiyatlarini ifodalaydi. Parametr  $a$  suyuqlik tezligining statsionar bo'lmagan o'zgarishini tavsiflaydi [3].

Ma'lumki, diffuziya koeffitsientlari (gidrodinamik dispersiya) suyuqlikning tezligiga bog'liq. Bu erda quyidagi bog'liqlik qabul qilinadi

$$D_x(x, t) = D_{x0} f_2(mt)(1 + ax)^2, \quad D_y(y, t) = D_{y0} f_2(mt)(1 + by)^2. \quad (6)$$

Keling, ikki o'lchovli g'ovak muhitda moddalarning anomal ko'chishini ko'rib chiqaylik. Biroq, agar muhit fraktal tuzilishga ega bo'lsa, ko'chish jarayoni anomal xarakterga ega bo'lib, uni kasr tartibli differentsial tenglamalar bilan modellashtirish mumkin [2].

Fazoviy koordinatalarga nisbatan anomal diffuziya hisobga olinsa, ko'chish tenglamasi quyidagicha yozilishi mumkin.

$$\begin{aligned} \frac{\partial C(x, y, t)}{\partial t} = & \frac{\partial^{\beta_1}}{\partial x} \left( D_x(x, t) \frac{\partial C(x, y, t)}{\partial x} - u(x, t) C(x, y, t) \right) + \\ & + \frac{\partial^{\beta_2}}{\partial y} \left( D_y(y, t) \frac{\partial C(x, y, t)}{\partial y} - v(y, t) C(x, y, t) \right), \end{aligned} \quad (7)$$

bu yerda  $0 < \beta_1 \leq 1$ ;  $0 < \beta_2 \leq 1$ .

(7) tenglama quyidagicha qayta yoziladi

$$\begin{aligned} \frac{\partial C(x, y, t)}{\partial t} = & \frac{\partial^{\beta_1} D_x(x, t)}{\partial x^{\beta_1}} \cdot \frac{\partial C(x, y, t)}{\partial x} + D_x(x, t) \cdot \frac{\partial^{1+\beta_1} C(x, y, t)}{\partial x^{1+\beta_1}} - \\ & - \frac{\partial^{\beta_1} u(x, t)}{\partial x^{\beta_1}} \cdot C(x, y, t) - u(x, t) \cdot \frac{\partial^{\beta_1} C(x, y, t)}{\partial x^{\beta_1}} + \frac{\partial^{\beta_2} D_y(y, t)}{\partial y^{\beta_2}} \cdot \frac{\partial C(x, y, t)}{\partial y} + \\ & + D_y(y, t) \cdot \frac{\partial^{1+\beta_2} C(x, y, t)}{\partial y^{1+\beta_2}} - \frac{\partial^{\beta_2} v(y, t)}{\partial y^{\beta_2}} - v(y, t) \cdot \frac{\partial^{\beta_2} C(x, y, t)}{\partial y}. \end{aligned} \quad (8)$$

Caputo ta'rifi quyidagicha bo'ladi

$$D_*^\alpha = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, & n-1 < \alpha < n \in N, \\ \frac{d^n}{dt^n} f(t), & \alpha = n \in N, \end{cases}$$

bu yerda  $\Gamma(\cdot)$ -gamma funksiyasi .

(8) yechish uchun oshkormas chekli ayirma sxemasidan foydalanish mumkin. Biroq, bu sxemadan foydalanish kasr tartibli hosilalar uchun juda murakkab hisoblanadi. Shuning uchun bu erda biz quyidagi shaklning oshkor ayirmali sxemasidan foydalanamiz [4].

$$\begin{aligned} \frac{C_{i,j}^{k+1} - C_{i,j}^k}{\tau} &= \frac{(D_x)_i^k - \beta_1 (D_x)_{i-1}^k}{\Gamma(2-\beta_1) h_1^{\beta_1}} \cdot \frac{C_{i,j}^k - C_{i-1,j}^k}{h_1} + (D_x)_i^k \cdot \frac{1}{\Gamma(3-\gamma_1) \cdot h_1^{\gamma_1}} \cdot \\ &\sum_{l=0}^{i-1} \left[ (C_{i+1-l,j}^k - 2 \cdot C_{i-l,j}^k + C_{i-1-l,j}^k) \times ((l+1)^{2-\gamma_1} - l^{2-\gamma_1}) \right] - C_{i,j}^k \cdot \frac{u_i^k - \beta_1 u_{i-1}^k}{\Gamma(2-\beta_1) \cdot h_1^{\beta_1}} - \\ &- u_i^k \cdot \frac{C_{i,j}^k - \beta_1 \cdot C_{i-1,j}^k}{\Gamma(2-\beta_1) \cdot h_1^{\beta_1}} + \frac{(D_y)_j^k - \beta_2 \cdot (D_y)_{j-1}^k}{\Gamma(2-\beta_2) \cdot h_2^{\beta_2}} \cdot \frac{C_{i,j}^k - C_{i,j-1}^k}{h_2} + \frac{(D_y)_j^k}{\Gamma(3-\gamma_2) \cdot h_2^{\gamma_2}} \cdot \\ &\sum_{l=0}^{j-1} \left[ (C_{i,j+1-l}^k - 2 \cdot C_{i,j-l}^k + C_{i,j-1-l}^k) \times ((l+1)^{2-\gamma_1} - l^{2-\gamma_1}) \right] - C_{i,j}^k \cdot \frac{v_j^k - \beta_2 v_{j-1}^k}{\Gamma(2-\beta_2) \cdot h_2^{\beta_2}} - \\ &- v_j^k \frac{C_{i,j}^k - \beta_2 \cdot C_{i,j-1}^k}{\Gamma(2-\beta_2) \cdot h_2^{\beta_2}}. \end{aligned} \quad (9)$$

(9) tenglama shaklda yoziladi

$$\begin{aligned} C_{i,j}^{k+1} &= \frac{\tau \cdot ((D_x)_i^k - \beta_1 (D_x)_{i-1}^k) \cdot (C_{i,j}^k - C_{i-1,j}^k)}{h_1^{1+\beta_1} \cdot \Gamma(2-\beta_1)} + \frac{\tau \cdot (D_x)_i^k}{\Gamma(3-\gamma_1) \cdot h_1^{\gamma_1}} \cdot \\ &\cdot \sum_{l=0}^{i-1} \left[ (C_{i+1-l,j}^k - 2 \cdot C_{i-l,j}^k + C_{i-1-l,j}^k) \times ((l+1)^{2-\gamma_1} - l^{2-\gamma_1}) \right] - \frac{\tau \cdot C_{i,j}^k (u_i^k - \beta_1 u_{i-1}^k)}{\Gamma(2-\beta_1) \cdot h_1^{\beta_1}} - \\ &- \frac{\tau \cdot u_i^k (C_{i,j}^k - \beta_1 \cdot C_{i-1,j}^k)}{\Gamma(2-\beta_1) \cdot h_1^{\beta_1}} + \frac{\tau \cdot ((D_y)_j^k - \beta_2 \cdot (D_y)_{j-1}^k) \cdot (C_{i,j}^k - C_{i,j-1}^k)}{\Gamma(2-\beta_2) \cdot h_2^{1+\beta_2}} + \\ &+ \frac{\tau \cdot (D_y)_j^k}{\Gamma(3-\gamma_2) \cdot h_2^{\gamma_2}} \cdot \sum_{l=0}^{j-1} \left[ (C_{i,j+1-l}^k - 2 \cdot C_{i,j-l}^k + C_{i,j-1-l}^k) \times ((l+1)^{2-\gamma_1} - l^{2-\gamma_1}) \right] - \\ &- \tau \cdot C_{i,j}^k \cdot \frac{v_j^k - \beta_2 v_{j-1}^k}{\Gamma(2-\beta_2) \cdot h_2^{\beta_2}} - \tau \cdot v_j^k \frac{C_{i,j}^k - \beta_2 \cdot C_{i,j-1}^k}{\Gamma(2-\beta_2) \cdot h_2^{\beta_2}} + C_{i,j}^k. \end{aligned} \quad (10)$$

Boshlang`ich shart quyidagicha chekli ayirmalar usuli bilan almashtiriladi

$$C_{i,j}^0 = 0, \quad x \geq 0; \quad y \geq 0; \quad t = 0 \quad (11)$$

Chegaraviy shartlar esa quyidagicha approksimatsiya qilinadi

$$C_{0,j}^{k+1} = \frac{\tau \cdot \left( (D_x)_1^k - \beta_1 (D_x)_0^k \right) \cdot (C_{1,j}^k - C_{0,j}^k)}{h_1^{1+\beta_1} \cdot \Gamma(2 - \beta_1)} - \frac{\tau \cdot C_{0,j}^k (u_1^k - \beta_1 u_0^k)}{\Gamma(2 - \beta_1) \cdot h_1^{\beta_1}} +$$

$$+ \frac{\tau \cdot \left( (D_y)_j^k - \beta_2 \cdot (D_y)_{j-1}^k \right) \cdot (C_{0,j}^k - C_{0,j-1}^k)}{\Gamma(2 - \beta_2) \cdot h_2^{1+\beta_2}} + \frac{\tau \cdot (D_y)_j^k}{\Gamma(3 - \gamma_2) \cdot h_2^{\gamma_2}} \cdot \quad (12)$$

$$\sum_{l=0}^{j-1} \left[ (C_{0,j+1-l}^k - 2 \cdot C_{0,j-l}^k + C_{0,j-1-l}^k) \times \left( (l+1)^{2-\gamma_1} - l^{2-\gamma_1} \right) \right] -$$

$$- \tau \cdot C_{0,j}^k \cdot \frac{v_j^k - \beta_2 v_{j-1}^k}{\Gamma(2 - \beta_2) \cdot h_2^{\beta_2}} - \tau \cdot v_j^k \frac{C_{0,j}^k - \beta_2 \cdot C_{0,j-1}^k}{\Gamma(2 - \beta_2) \cdot h_2^{\beta_2}} + C_{0,j}^k,$$

$$C_{i,0}^{k+1} = \frac{\tau \cdot \left( (D_x)_i^k - \beta_1 (D_x)_{i-1}^k \right) \cdot (C_{i,0}^k - C_{i-1,0}^k)}{h_1^{1+\beta_1} \cdot \Gamma(2 - \beta_1)} + \frac{\tau \cdot (D_x)_i^k}{\Gamma(3 - \gamma_1) \cdot h_1^{\gamma_1}} \cdot$$

$$\cdot \sum_{l=0}^{i-1} \left[ (C_{i+1-l,0}^k - 2 \cdot C_{i-l,0}^k + C_{i-1-l,0}^k) \times \left( (l+1)^{2-\gamma_1} - l^{2-\gamma_1} \right) \right] -$$

$$- \frac{\tau \cdot C_{i,0}^k (u_i^k - \beta_1 u_{i-1}^k)}{\Gamma(2 - \beta_1) \cdot h_1^{\beta_1}} - \frac{\tau \cdot u_i^k (C_{i,0}^k - \beta_1 \cdot C_{i-1,0}^k)}{\Gamma(2 - \beta_1) \cdot h_1^{\beta_1}} +$$

$$+ \frac{\tau \cdot \left( (D_y)_1^k - \beta_2 \cdot (D_y)_0^k \right) \cdot (C_{i,1}^k - C_{i,0}^k)}{\Gamma(2 - \beta_2) \cdot h_2^{1+\beta_2}} - \tau \cdot C_{i,1}^k \cdot \frac{v_1^k - \beta_2 v_0^k}{\Gamma(2 - \beta_2) \cdot h_2^{\beta_2}} + C_{i,j}^k \cdot \quad (13)$$

Ko'rinib turibdiki, hosila tartibi  $\beta_1$  ni1 dan kamayishi bilan konsentratsiyaning taqsimlanishi *o'qi* yo'nalishlarida ortishi ko`rinadi .

$\beta_1$  va  $\beta_2$  qiymatlarni 1 dan kamaytirish "tez diffuziya" ga olib keladi, konsentratsiya profillari maydon bo'ylab yanada intensiv tarqaladi. Shunday qilib, agar  $x$  kordinata  $y = 0$   $\beta_1 = \beta_2 = 1$  qiymatlarda konsentratsiya maydoni taxminan  $x = 0,9$  m chegarasiga yetsa,  $\beta_1 = \beta_2 = 0,9$  da chegara  $\sim 1,25$  m ga yetadi,  $\beta_1 = \beta_2 = 0,7$  holda esa bu chegara  $\sim 1,5$  m ga yetadi. Bundan ko`rinadiki hosila tertibi 1 dan kamayishi konsentratsiya profillarining kengroq yoyilishiga olib keladi. Shunday qilib, moddalarning kasr hosilasi shaklida koordinatali yo'nalishlarda anomal ko'chishi "tez diffuziya" ga olib keladi.

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