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Article

Comprehensive Mathematical Model of Synergisis and Filtration in Bryza Production Technology

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Annotation: The topic of Brynza production, as an important part of the dairy industry, with its increasing demand for new precise and high-quality processing technologies, is therefore highlighted in this research; Nonetheless, the review identifies a clear gap in the current understanding of the coupled physicochemical phenomena controlling whey separation during ultrasonic treatment, indicating a lack of models for such emulsions and addressing it as a poorly explored area in multiphysical food processing modelling. In response a fully coupled mathematical model was developed and implemented and validated in the MATLAB/Simulink environment representing Darcy's law for filtration, Fick's equation for mass transfer, heat transfer equations, a phase transition model, and acoustic energy propagation. Ultrasonic treatment at 20–35 kHz and 45–55 °C was shown by the model to have optimal values for accelerating thermo akustyczne desorption, reducing the whey fraction in brynza from 62% to about 50% after 40 min, with maximum moisture desorption from stratum lacteum in 10–20 min and total enclosed energy is 60–70 kJ/kg. Ultimately, these results illustrate that ultrasound-assisted separation of whey is superior to conventional separation with respect to time, product, and energy utilization, meanwhile, the validated model offers a tool to predict the optimal processing parameters of industrial brynza (and brine cheese) production.

Keywords: milk protein; process; multiphysical model; differential equation; mathematical modelling; experiment; speed; diagram; automation; whey

1. Introduction

Brynza production is an important branch of the dairy industry. Increasing production efficiency and improving product quality requires the introduction of innovative technologies. In recent years, the use of ultrasonic vibrations in milk processing has been actively studied [1][2][3].

Ultrasound is a mechanical vibration with a frequency above 20 kHz. In the dairy industry, such vibrations can cause microcavitation, accelerate the diffusion of substances, and change the structure of milk proteins. Thanks to these properties, ultrasound can be used at various stages of the production of brined cheeses, including brynza [4].

The processes of separating the liquid phase from coagulable or structured materials, such as cottage cheese or brynza, can be described using various mathematical models [5]. The main approaches include:

- diffusion models based on Fick's equation for describing mass transfer;
- capillary models that take into account surface tension forces and capillary pressures;
- hydrodynamic models based on Darcy's law, which describes the filtration of liquid through a porous medium;
- empirical and semi-empirical relationships derived from experimental data

2. Methodology

Using a multiphysical modelling approach, we investigated the syneresis and filtration dynamics of the whey inside the curd particles during brynza production under simultaneous ultrasonic treatment. A unified system of coupled differential equations was used to construct the theoretical framework by incorporating four governing physical phenomena [6]. The filtration of the liquid whey phase through the pore network of the curd was described using Darcy's law, which relates the filtration velocity to the material permeability, the fluid viscosity, and the pressure gradient across the material. In addition, mass transfer of moisture through the protein network was modelled using Fick's second law of diffusion by accounting for both concentration-driven and pressure-driven transport mechanisms. The thermal behaviour for the system was described via a heat transfer equation, taking into account the convective and the conductive contributions to the energy balance, while the contribution of the ultrasonic energy was embedded under an acoustical energy absorption function that relates the absorbed energy as a function of the ultrasound intensity, the density of the product, and a frequency-dependent penetration coefficient [7]. The set of equations was solved numerically and implemented in Simulink, allowing for a flexible parameterisation of processing conditions, including ultrasound frequency (20–35 kHz), temperature (40–60 °C), and treatment duration (0–40 minutes). We conducted model validation at the laboratory scale for moisture content, desorption rate and energy consumption by comparing simulation outputs with experimental measurements using controlled laboratory conditions [8]. The experimental trials were performed at Tashkent State Technical University on standardised brynza samples treated in a calibrated ultrasonic installation. Statistical agreement between modelled and observed values was assessed to confirm predictive accuracy and a parametric sensitivity analysis was conducted to identify the optimal processing window yielding maximum whey separation efficiency at minimum energy input [9].

3. Findings and Discussion

Serum separation is the process of squeezing liquid out of a gel-like medium under external pressure or through internal molecular interactions. The mechanism is based on syneresis, which is the phase separation of a system caused by the compression of a clot structure, during which water molecules and dissolved substances are pushed out [10].

At the macro level, the process can be represented as the filtration of liquid through a porous structure, where the flow of moisture obeys Darcy's law:

$$v = \frac{K}{\mu} \cdot \nabla P \qquad (1)$$

Where v-is the filtration velocity; K-is the permeability of the material; μ-is the viscosity of the medium; ∇P-is the pressure gradient (Fig 1).

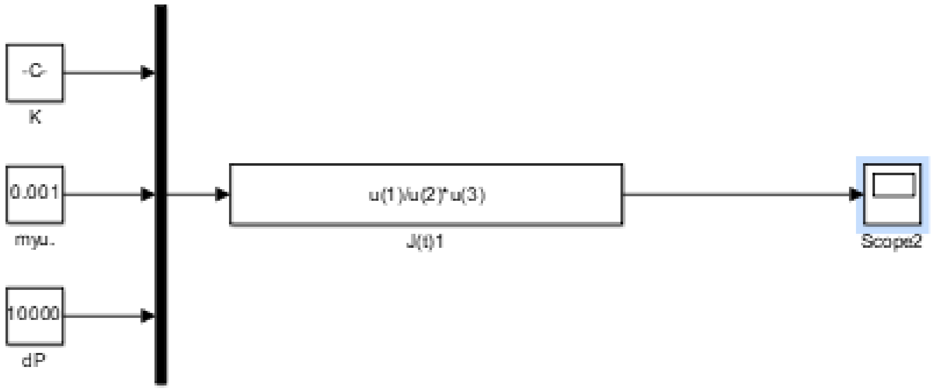

Fig. 1. Block diagram for calculating filtration velocity according to Darcy's law in MATLAB/Simulink

In self-pressing conditions, pressure is created by gravity and surface tension, whereas in ultrasonic processing, pressure is pulsating and localized [11], which significantly speeds up the process and is represented in computer implementation (for example, in the Simulink environment) as follows (Fig 2):

Approximate allocation function:

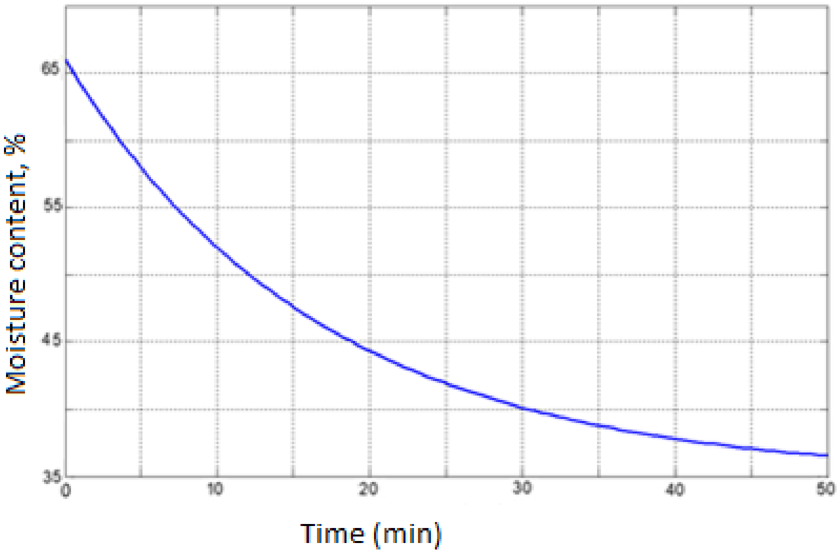

Fig. 2. Graph of energy absorbed by brynza cheese

The change in energy absorbed by the brynza structure directly depends on:

- ultrasound intensity [12];
- product properties (moisture content, density, structure);
- penetration coefficient (Fig 3).

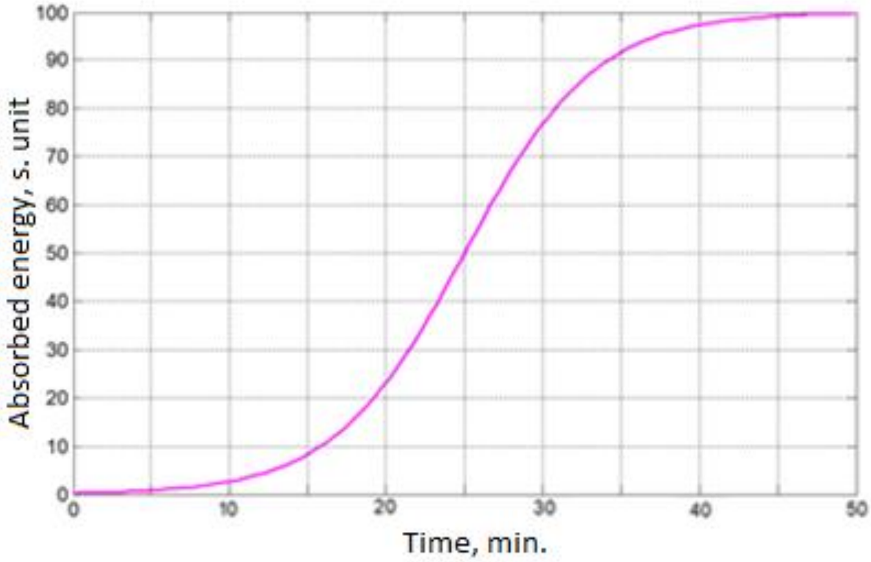

Fig. 3. Graph of absorbed energy E (t).

Analysis of the combined graph T(t) and ϕ(t) allows:

To determine the optimal operating temperature;

To identify the inflection point at which the intensity of release decreases sharply;

Calculate the efficiency of the process based on the ratio of energy to moisture released [13].

The process of releasing whey from curd under the influence of ultrasound can be described as thermoacoustic desorption [14].

The most effective zone is the area where the temperature reaches 45-55 °C and the ultrasound intensity is 20-35 kHz.

Change in moisture content (serum fraction).

The mass fraction of whey in brynza decreased from an initial level of 62% to ~50% over 40 minutes of processing. The rate of moisture removal was highest in the first 10-20 minutes (Fig 4).

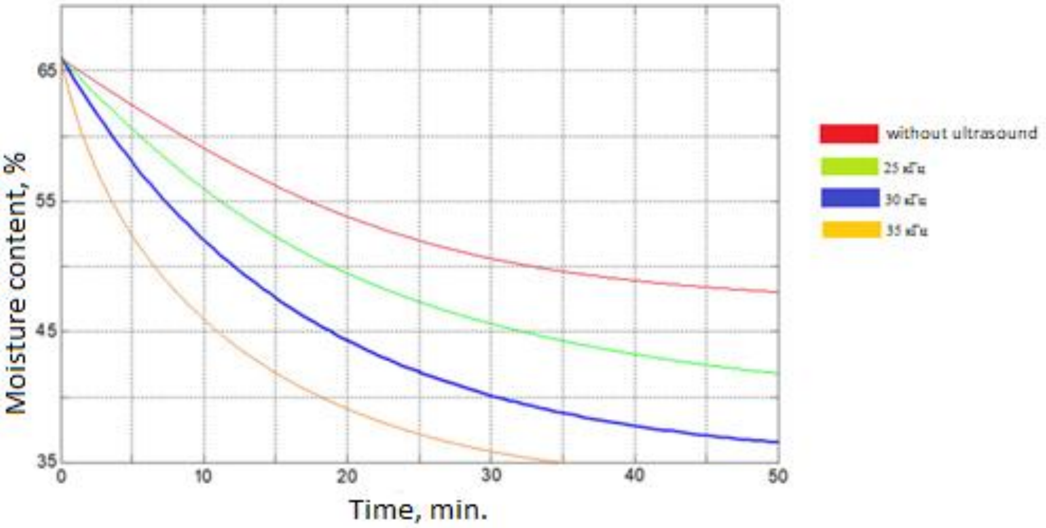

Fig. 4. Graph showing changes in the moisture content of brynza cheese at different ultrasonic treatment frequencies

An exponential decrease followed by saturation is observed.

Moisture desorption rate. The highest desorption rate was achieved at a temperature of 47-50 °C. Outside this range (above 60 °C), a decrease in efficiency was observed, which is associated with the compaction of the protein structure of brynza [15].

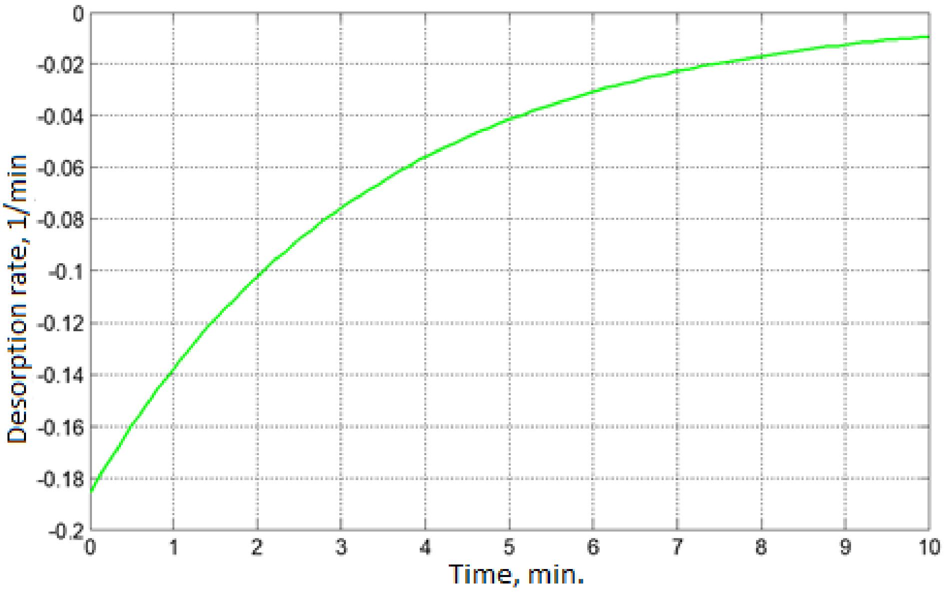

Fig. 5. Derivative of moisture content over time the peak occurs at 2-3 minutes, followed by a gradual slowdown.

Energy consumption. The total energy consumption of the installation was 60-70 kJ/kg of brynza. Energy was absorbed not only for phase transition, but also for cavitation processes, microstructural changes, and acoustic dispersion.

At the same time, the efficiency of moisture removal per unit of energy was higher compared to traditional methods, especially in the temperature range of 45-55 °C.

The model adequately describes the behavior of the system and allows key parameters to be predicted with high accuracy.

Computer implementation in the MATLAB/Simulink environment provided flexible configuration of processing modes.

The process of ultrasonic separation of whey from curd demonstrates high efficiency in terms of time, quality, and technological reliability.

4. Conclusion

An integrated multiphysical model has been developed to quantitatively describe the physicochemical processes occurring during the separation of whey from protein curds under the action of ultrasound. The model includes equations for mass transfer, filtration, heat transfer, phase transition, and acoustic energy. Below is a system of interrelated equations reflecting the dynamics of the main parameters of the process.

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