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Improvement of a Robotic Manipulator Model Based on Multivariate Residual Modeling

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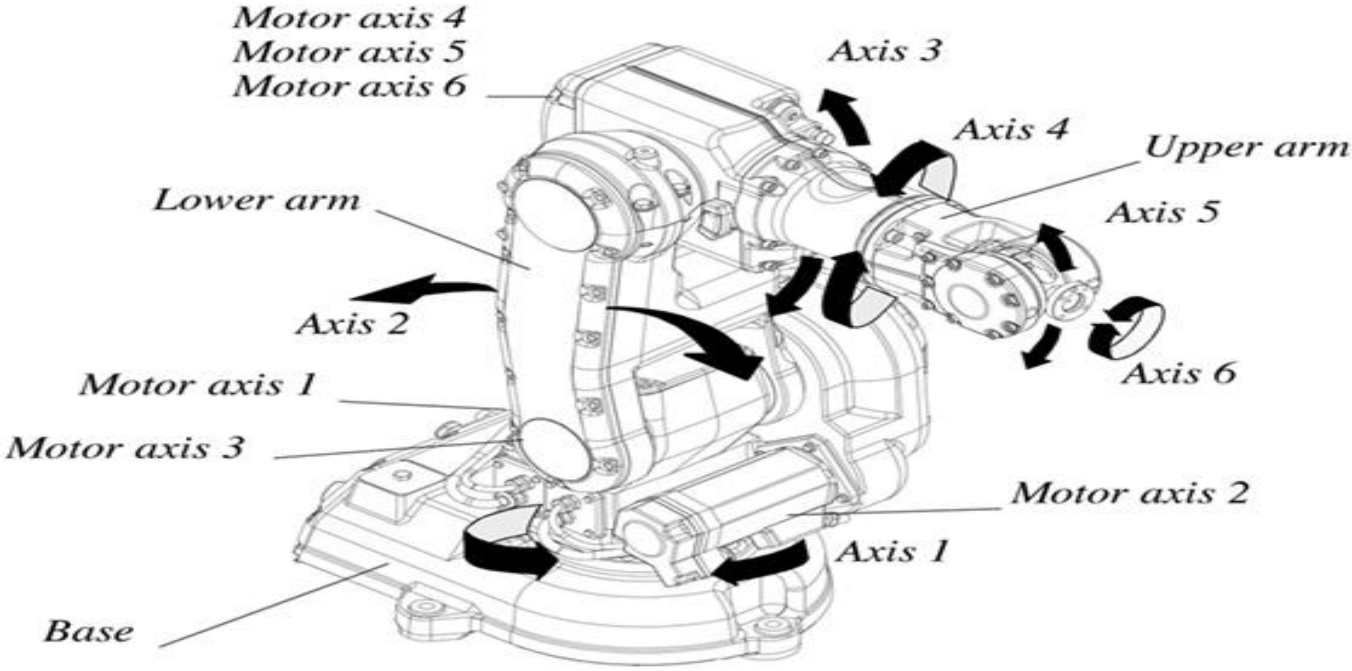
Annotation: The performance of the new method is compared with least squares (LS). Different cross-validation schemes were compared in order to assess the sampling of the state space based on conventional trajectories. The method developed in the paper can be used as fault monitoring mechanism and early warning system for sensor failure.

Keywords: Method, dynamic, degrees, performance, sampling, sensor failure, manipulator, interactions, position, principal, mechanistic model.

A new method is presented for extending a dynamic model of a six degrees of freedom robotic manipulator. A non-linear multivariate calibration of input–output training data from several typical motion trajectories is carried out with the aim of predicting the model systematic output error at time ($t + 1$) from known input reference up till and including time (t). A new partial least squares regression (PLSR) based method, nominal PLSR with interactions was developed and used to handle, unmodelled non-linearities. The performance of the new method is compared with least squares (LS). Different cross-validation schemes were compared in order to assess the sampling of the state space based on conventional trajectories. The method developed in the paper can be used as fault monitoring mechanism and early warning system for sensor failure. The results show that the suggested methods improves trajectory tracking performance of the robotic manipulator by extending the initial dynamic model of the manipulator.

1. Introduction

Control of various mechanical systems, such as robotic manipulators, autonomous ground vehicles (AGV), unmanned aerial vehicles (UAV), and surface vehicles (USV), require good model knowledge for precise and efficient control. It has been shown that model-based control is superior to non-model-based equivalent, this, however, requires rigorous mathematical modeling and detailed system analysis in order to develop a good and representative model of the system under consideration. In some cases, a dynamic model can be simple, linear, single input single output (SISO) system, such as a pendulum or mass on a spring [this does not, however, mean that a SISO system is less non-linear than its multiple inputs and outputs (MIMO) equivalent, in fact there is no direct correlation between complexity of the system in terms of non-linearities and its number of inputs and/or outputs]; in others, it may contain many degrees of freedom with MIMO, and have numerous sources of non-linear behavior such as a robotic manipulator, as shown in Figure 1. In the case of a standard industrial 6-DOF manipulator such as ABB IBR140 or KUKA KR150, non-linearities come from multiple sources, some are taken into consideration during model development stages following Lagrangian formulation (Spong et al., 2006; Siciliano et al., 2009). The result of systematic approach in developing a dynamic model is a differential equation governing the motion of a system. For a robotic manipulator, the dynamic model defines the relationship between joint position q_i, angular velocity ˙qiq˙i, and angular acceleration ¨qiq¨i to torque τ_i necessary to achieve desired position, velocity, and acceleration.



However, deductive, first principle models of complex mechanical, chemical, or biological systems lack real world touch. Relying on an incomplete model in general may result in system instability and poor performance. In control engineering, adaptive algorithms allow detrimental effects of unmodelled dynamics to be more or less neutralized over time. However, while we let the computational system correct for the mistakes automatically by the means of feedback, we do not learn what was wrong with the model initially and hence do not correct the original model inductively. Still, this is highly successful. However, with an incomplete dynamic model, the controller needs longer time to discover what is going on in the system to correct it and in some cases it may not be able to do so at all.

In this paper, we focus on investigating the properties of the error signal generated by the internal controller of an industrial robotic manipulator and a model of the same system based on Euler–Lagrange formulation combined with dynamic parameter identification procedure. The discrepancies between two systems, assuming zero measurement noise and ideal conditions is, therefore, the unmodelled dynamics in the theoretical model. To improve the theoretical model, two methods are used: principal component analysis (PCA) and partial least squares regression (PLSR) (Wold, 1985; Helland et al., 1992). PCA is used to test for and reveal structure in the error between the two models, while PLSR is used to modify the initial dynamic equation describing the system. The focus of this particular study is not to improve the output of the real system directly, however, but rather by developing a method by which a more accurate model of the real system can be achieved.

This present paper is a part of an ongoing effort to combine the best of the inductive and the deductive cultures. It has been shown that when a seriously erroneous or incomplete mathematical model is fitted to empirical data, the estimated model parameters may have alias errors. In Martens (2011), the author showed that multivariate subspace modeling of the high-dimensional residuals between measurements and model predictions could give surprisingly detailed quantification of unexpected and thus unmodelled phenomena in the system. It is our goal to improve understanding and mechanistic modeling of a real world system from more in-depth analysis of the residuals between models and measurements. Figures 2, 3, and 7 outline this general approach and intend to give better models, better understanding, and better process control.

FIGURE 2

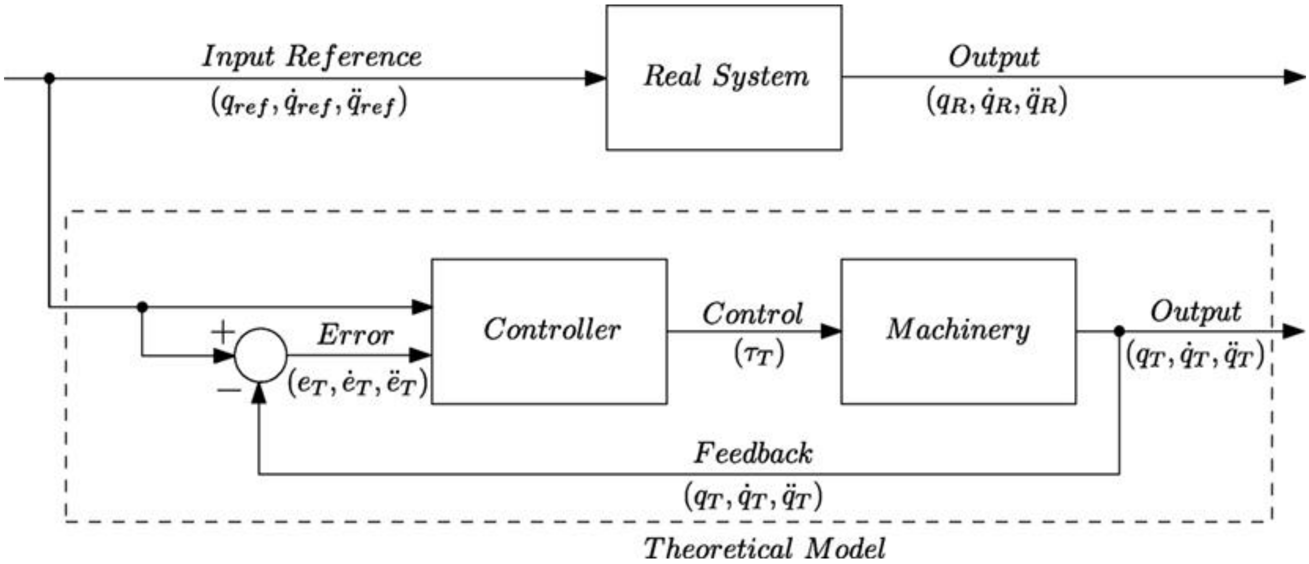
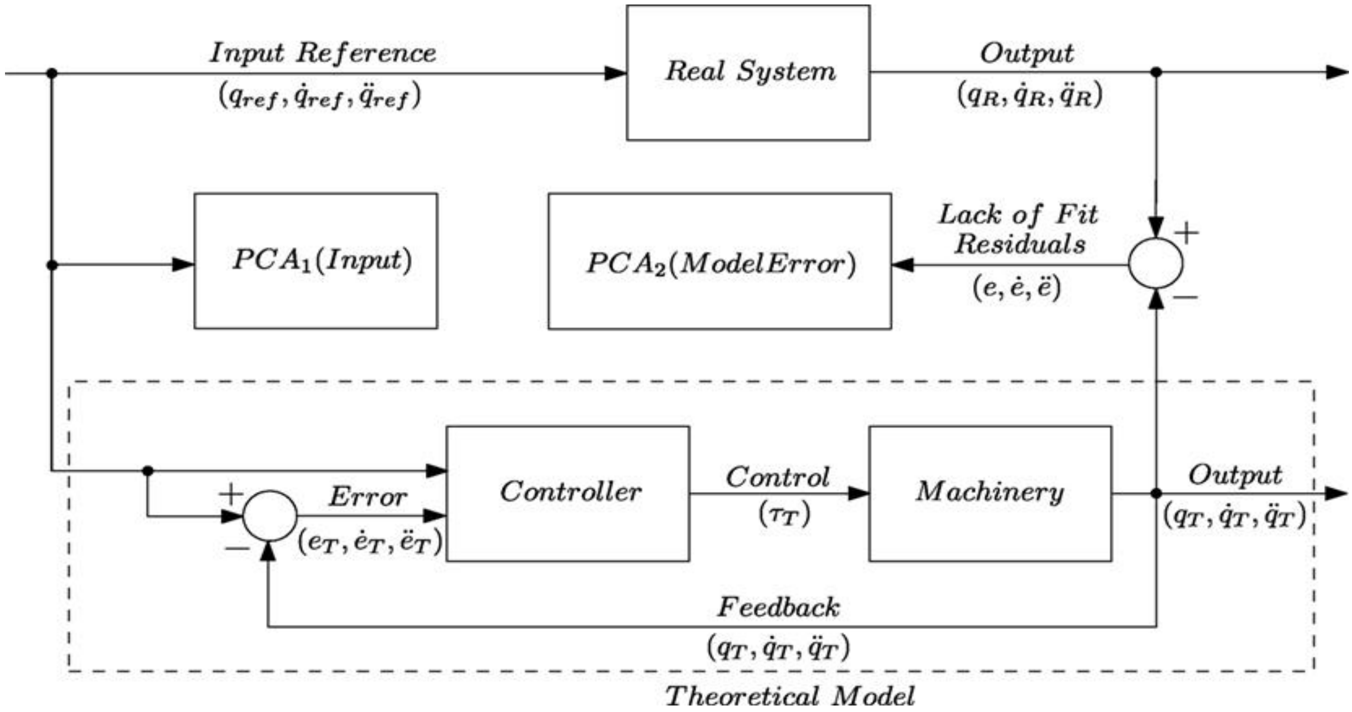


FIGURE 3



The first part of this paper outlines in detail the process of developing dynamic model for a rigid body based on first principles described by the basic laws of motion and conservation of momentum, while the second part attempts to improve the quality of the theoretically derived model based on gray box methodology. The initial method is then extended to include data-driven approach as a part of model improvement based on statistical methods.

There are several ways in which the theoretical and data-driven modeling can be combined. Particularly, we focus on using multivariate calibration tools to correct for the errors in the outputs from a mechanistic model. The goal is to improve the estimation of the torque τ needed to control the robotic arm to follow various predetermined trajectories via joint space control. We achieve that by modeling the observed lack-of-fit residual **Y** between mechanistic model predictions of torque and actual measurements of the *"true"* torque $\tau*\tau*$ from the desired trajectory specified as position **q**, velocity ˙qq˙, and acceleration ¨qq¨ in task space **X** by a subspace regression methods (Wold, 1985).

The mapping between **X** and **Y** is highly non-linear; therefore, we introduce a new version of the PLSR method (PLSR with nominal level representation of the **X**-variables). This is an extension of the nominal level PLSR used by Martens (2009), in the sense that not only main-effects but also interaction effects are modeled in the nominal level PLSR.

1.1. Dynamic Model Development

For an industrial manipulator as shown in Figure 1, the dynamic model tends to exclude effects, such as friction, gear oil viscosity, and shaft torsion compliance, etc. It is possible to determine some of those effects experimentally, for example a friction model; however, the validity of the model is only over a limited range of motion and conditions, none the less it appears that manufacturers tend to include this in their product. Generally external effects and unmodelled dynamics is treated as an undesirable disturbance and remedied via high gain control action or continuous adaptation in the case of adaptive control strategies.

A definition for a rigid body can be formulated such as a system of particles, which are subjected to some constraints, e.g., holonomic equation (1) where distances between all of the particles remain constant during motion (Goldstein, 1980). The principle of holonomic constraints can be given in a short form as:

f(v1,v2,...vn,t)=0,

where v=dx⁄dtv=dx⁄dt, a first derivative of a distance vector **x** of a particle from some given origin.

One way to visualize a rigid body subjected to holonomic constraints is to imagine a mass that is restrained to move along a predefined path, such as a stiff wire or a double pendulum with masses attached to each of the constant length rigid rods. The issue of the constraints is that the coordinates of the body is no longer independent and the forces acting on the particle because of the constraints are not known a priory (Finn, 2009). The solution to holonomic constraints comes from introduction of a generalized set of coordinates. A system with n particles free to move in all three dimensions is said to have $3n$ independent degrees of freedom; however, if the system is subject to k holonomic constraints the system is, therefore, reduced to $3n - k$ independent coordinates (Goldstein, 1980; Spong et al., 2006).

The process of deriving the dynamic model is mathematically involved and relies upon a number of well-known laws and principles of classical mechanics. There are two commonly used approaches to this. The first is the energy based approach and known as Euler–Lagrange formulation, which derived from the principle of virtual work. This formulation has a number of attractive properties for analysis of feedback control system, such as skew symmetry and explicit bounds on the inertia matrix as well as linearity in the inertial parameters. The method is well suited for developing control strategies based on energy and passivity principles (Slotine and Li, 1991; Khalil, 2000). An alternative to the Euler–Lagrange approach is Newton–Euler formulation. The latter method is a recursive formulation for rigid body dynamics and is more suitable for numeric calculations. The Newton–Euler formulation is well suited for real time inverse dynamics calculation and is very well suited for model based control system implementation. The complete derivation procedure is outside of the scope of this paper and will be omitted; however, detailed descriptions are given in Egeland and Gravdahl (2002), Craig (2005), Spong et al. (2006), and Siciliano et al. (2009).

The aim of developing a dynamic model of a rigid body or system of rigid bodies is to derive a set of differential equations that govern time evolution of the systems which is subject to a set of constraints. Systems, such as double pendulum, mass-spring-damper, or a robotic manipulator, are subject to holonomic constraints.

Conclusion

In order to develop a dynamic model, it is necessary to analyze and derive kinematics of a solid object that describes position and velocity of the body in space. As it has been mentioned before, once a set of independent coordinates has been specified, it is then possible to start developing body kinematics based on vector algebra. A rigid body can be described by six independent coordinates, three for position and three for orientation. The transformation from chosen fixed coordinate system in space to a fixed coordinate system attached to a rigid body is known as an orthogonal transformation. A rotation matrix **R** that fulfils orthogonality conditions $r_{ij}r_{ik} = \delta_{jk}$ is called orthogonal and has a number of useful properties, such as $\mathbf{R}^T\mathbf{R} = \mathbf{I}$, where **I** is the identity matrix, and $\mathbf{R}^T = \mathbf{R}^{-1}$ (Spong et al., 2006; Siciliano et al., 2009). The elementary rotation of a rigid body by some angle in space can be represented as an independent rotation along each of the axis in turn. Composition of rotations is achieved via pre or post multiplication of rotation matrices with coordinate systems attached to intermediate frames of reference. For a two dimensional rotation, the **R** is a two by two matrix, for a three dimensional rotation the **R** is a three by three matrix, defining nine r_{ij} not independent directional cosines. A set of all $m \times m$ orthogonal matrices is referred to a Special Orthogonal group of order m and is denoted $SO(m)$ (Khalil and Dombre, 2004; Craig, 2005). The rotation matrix can be parametrized in a number of ways, including Euler angles and quaternions. Rotational transformation only describes rotation of one frame with respect to the other, combination of rotation and translation in a single matrix **H** defines the homogeneous transformation [equation (2)],

$$H = \begin{bmatrix} \mathbf{R} & \mathbf{d} \\ 0 & 1 \end{bmatrix},$$

where **R** is 3 × 3 rotation matrix, **d** is 3 × 1 translation vector.

Once kinematics of a rigid body has been established the forward and inverse kinematic chains for a multi-link robotic manipulator can be developed. Forward and inverse kinematics are seen as a map between two coordinate systems, i.e., joint space vs task space, where the latter is the inverse of the former. The motion of a rigid body through space gives rise to velocity kinematics, which defines description for linear and angular motion. For a robotic manipulator velocity, kinematics provides solution to two types of joint: revolute and prismatic. This in turn defines a manipulator unique Jacobean matrix, which relates linear and angular velocities of the end effector to individual joint velocities.

Armed with the above knowledge manipulator dynamic equation is defined as:

M(q)¨q+C(q,˙q)˙q+G(q)+F(˙q)=τ,M(q)q¨+C(q,q˙)q˙+G(q)+F(q˙)=τ, (3)

where M(q)∈Rm×nM(q) ∈ Rm×n is a positive definite inertia matrix, C(q,˙q)∈Rm×nC(q,q˙) ∈Rm×n is centrifugal and Coriolis forces matrix, G(q)∈Rm×1G(q) ∈ Rm×1 is gravitation vector, F(˙q)∈Rm×1F(q˙) ∈ Rm×1 is friction vector, q,˙q,¨q∈Rm×1q,q˙,q¨ ∈ Rm×1 are joint angles, velocity, and acceleration vectors, respectively, and τ∈Rm×1τ ∈ Rm×1 is a vector of actuator torques. Note that the inertia matrix **M**, centrifugal and Coriolis matrix **C**, gravity vector **G**, and friction matrix **F** are non-linear functions.

The structure of the dynamics in equation (3) is not unique and is also observed in many other mechanical systems such as aerial or ground vehicles and marine vessels.

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