



# International Conference of Economics, Finance and Accounting Studies

*International Conference of Economics, Finance and Accounting Studies is a double-blind peer-reviewed, open-access journal published to reach excellence on the scope. It considers scholarly, research-based articles on all aspects of economics, finance and accounting. As an international congress aimed at facilitating the global exchange of education theory, contributions from different educational systems and cultures are encouraged. It aims to provide a forum for all researchers, educators, educational policy-makers and planners to exchange invaluable ideas and resources.*

## Logarithmic Equations: Theoretical Foundations and Solution Methods

**Mamaraimov Bekzod Kadirovich, Makhmudov Azam Kudratovich,  
Musurmonov Maruf Akrom oglu**

Teacher of mathematics at Terdu Academic Lyceum

In mathematics, the use of equations is widespread in solving many problems and examples. In particular, many practical problems are solved through systems of equations, equations of the first, second or higher degree. Some complex problems encountered in life are solved directly using other mathematical branches, including logarithms. Especially in problems related to technology, physical processes or expressing the dynamics of growth and decay, logarithmic equations are an important tool.

Logarithmic equations come in various forms, and several methods are used to solve them. For example, there are problems ranging from simple logarithmic equations to complex polynomial logarithmic expressions. These methods include methods such as equating bases, using the properties of logarithms, and converting logarithmic expressions to exponential form, which simplify equations and find solutions.

The topic of logarithms is also reflected in the 10th grade “Mathematics” textbook of general education schools in Uzbekistan. Although this textbook provides logarithmic expressions, equations, and related examples, in some cases there is a lack of sufficient theoretical information and analysis on this topic. Therefore, additional materials and explanatory analyses are necessary to study this topic more deeply, to explain its scope of application in real life, to form students' interest in the topic and practical skills.

Logarithms are important not only theoretically, but also practically, and occupy a special place in the development of science and technology. Therefore, in-depth study of logarithmic equations and methods for solving them is considered one of the relevant and useful areas of mathematics.

Logarithms are one of the important concepts in mathematics. They originated as inverse functions of exponential functions and are widely used in various fields of science - physics, chemistry, economics, and engineering. In particular, solving logarithmic equations is of great practical importance in the fields of mathematical analysis and algebra.

A logarithmic equation is an equation in which an unknown number is included in the logarithm.

General appearance:

$$\text{Log}_a f(x) = g(x)$$

Here  $a > 0$ ,  $a \neq 1$ ,  $f(x) > 0$ , and  $g(x)$  can be any expression.

Main properties of logarithms:

- ✓  $\text{Log}_a(xy) = \log_a x + \log_a y$
- ✓  $\text{Log}_a(x/y) = \log_a x - \log_a y$
- ✓  $\text{Log}_a(x^n) = n \log_a x$

When solving logarithmic equations using graphs, it is first necessary to draw a graph of the logarithmic function. Logarithmic functions are either increasing or decreasing depending on their base (i.e., the value of  $a$ ). If the base is  $a > 1$ , the logarithmic function is increasing; conversely, if  $0 < a < 1$ , then the function is decreasing.

When drawing a graph, it is important to note the points at which the logarithmic function intersects the  $x$  and  $y$  axes, as well as the presence of vertical asymptotes. For a logarithmic function, the point  $x = 0$  always serves as a vertical asymptote, since the function is not defined at this point and tends to infinity as it approaches it.

The function shown on the other side of the equation is also depicted graphically. This other function can usually be linear ( $y=mx+b$ ) or quadratic ( $y=ax^2+bx+c$ ). This graph identifies the  $x$ - and  $y$ -intercepts, extrema (i.e., maximum or minimum points), and asymptotic behavior.

Once both functions are graphed, the points of intersection are found. The  $x$ -coordinates of these intersections are the actual solutions to the logarithmic equation. This graphical approach allows for a visual analysis of logarithmic equations and helps students understand the relationships between functions.

Solving logarithmic equations graphically is one of the effective approaches widely used in modern mathematics and analysis. Using this method, it is possible to determine how many intersection points the graph of a logarithmic function has with the graphs of other functions (for example, linear, quadratic, exponential or other logarithmic functions). Each intersection point is interpreted as a specific solution to the equation, respectively.

Many logarithmic equations may have not one, but several solutions. This is especially true when the graphs intersect several times. Therefore, each intersection point must be considered separately. In some cases, the equation may not have any real solution or, conversely, may have an infinite number of solutions. The graphical method is very convenient and intuitive in determining these cases.

Modern technological tools for solving logarithmic equations graphically - graphing calculators, special programs (GeoGebra, Desmos, WolframAlpha, etc.) are widely used. These tools allow you to draw graphs accurately and quickly, which helps save time and effort in determining solutions. In addition, with the help of these programs, you can analyze the main properties of graphs (asymptotes, domain of accuracy, interval of increase or decrease).

The graphical method is important not only in solving logarithmic equations, but also in a deeper study of mathematical functions in general. It allows students to visually display mathematical relationships, help them understand them, and gain solid knowledge through the images created when solving problems. The graphical approach also serves to increase student interest as an interactive and visual method in teaching mathematics.

That is, solving logarithmic equations graphically is not only a practical, but also an educationally useful approach, which plays an important role in developing mathematical thinking and simplifying the process of finding solutions.

The field of science related to calculating logarithms is called logarithmology. In practice, the base ( $a$ ) and argument ( $b$ ) of logarithms are often real numbers, but in mathematics, logarithms are also defined and used for complex numbers. Logarithms are famous for their unique properties, and their calculation allows you to simplify many complex arithmetic operations.

Therefore, logarithmic functions are widely used in science, technology, economics, and many other areas. Using logarithms, the operation of multiplying numbers is equated to simple addition, and division is equated to subtraction. Also, raising to a power is equated to multiplication with logarithmic operations, and taking a root is equated to division. These properties significantly speed up calculations. Therefore, in history, the creation of logarithms was an important turning point in the development of science. One of the great scientists, Laplace, said about logarithms: "Logarithms have lightened the work of the mathematician and doubled his life."

The theoretical foundations and tables of practical values of logarithms were first published in 1614 by the Scottish mathematician John Napier. This work was met with great interest by the scientific community and was later further developed and improved by other mathematicians. Logarithmic tables, especially in calculations related to trigonometric functions, have played an important role and have served as one of the main tools for mathematicians for many years. In engineering, logarithmic functions are important in calculating signal strength and developing control algorithms. In statistics, logarithmic transformations are used in statistical analysis and data normalization. Logarithmic functions are an important part of mathematical analysis, and knowing and understanding their properties is important in mathematical analysis and solving practical problems. The unique properties and practical applications of logarithmic functions make them an integral part of mathematics. Their application in many areas further increases their importance. By studying logarithmic functions, we can be more effective in mathematical analysis and solving practical problems.

In this way, logarithms have been widely used not only in mathematical theory, but also in real life, making a significant contribution to the development of humanity.<sup>1</sup>

In conclusion, solving logarithmic equations is an important practical skill in mathematics. To solve these equations, it is necessary to have a deep understanding of the properties of logarithms, pay attention to the domain of definition, and also know graphical analysis. Logarithmic equations are widely used in modeling many processes in real life.

## References

1. Aliyev Sh. Algebra va analiz asoslari. — Toshkent: O‘qituvchi, 2020.
2. Rasulov A. Matematika nazariyasi va amaliyoti. — Samarqand, 2019.
3. Stewart, J. (2016). Calculus: Early Transcendentals (8th ed.). Cengage Learning.
4. Karimov, A. (2018). “Matematika va uning amaliyotdagi o‘rni”. Toshkent: O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi.
5. Rahmonov, I. (2020). “Matematik analiz asoslari”. Samarqand: Samarqand Davlat Universiteti.
6. Tashkentov, M. (2019). “Logarifmik funksiyalar va ularning qo‘llanilishi”. Buxoro: Buxoro Davlat Universiteti.

---

<sup>1</sup> Tashkentov, M. (2019). “Logarifmik funksiyalar va ularning qo‘llanilishi”. Buxoro: Buxoro Davlat Universiteti.