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Article

Synthesis of Permutation Methods for Error-Correcting Codes in Noisy Channels

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Abstract: Methods for synthesizing error-correcting codes are examined in the context of communication channels subjected to additive noise through permutation-based methods. This research is motivated by the increase in interference on modern communication systems and the need for enhanced data transmission efficiency as a result of that interference. New algorithms are created for the construction of error-correcting codes based on the structural properties and combinatorial properties of permutations, with an objective to reduce the incidence of decoding errors. The efficacy of these newly developed methods has been assessed against the traditional methods of error-correcting codes, namely Reed-Solomon codes, and Convolutional codes, through a comparison of simulated data. According to results from simulations, increasing the amount of noise immunity can be accomplished by reducing the amount of relative errors by $18\% - 22\%$ in Bit Error Rate (BER) with $E_b/N_0 = 6\,dB$ (e.g. a reduction of $BER = 4.1 \times 10^{-2}$ with a Viterbi decoder to $BER = 3.2 \times 10^{-2}$ using a Structural Decoder and finally to $BER = 1.9 \times 10^{-2}$ with an ML decoder). With this we were also able to keep our transmission rate. All of the performance references were made in relation to the Shannon Limit.

The methods discussed above have been validated for code block lengths of up to $n = 64$ by using subgroup construction of the normal group S n and Galois Field mapping. However, we only tested the experimental simulations for $n = 4$ and $n = 8$ because of the $5\,ms$ real-time decoding requirements established in the 5G URLLC and IoT applications running on an ARM Cortex M4 processor. Our algebraic framework can be extended for use with code block lengths of $n = 16$, $n = 32$ and $n = 64$ as a result of hardware acceleration. In the future, researchers will use various algebraic techniques to optimize the order of permutations. This research provides insight that can aid in developing coding theories that may be applied in both 5G and IoT systems.

Keywords: error-correcting codes; error correction; noisy channels; permutation methods; transmission efficiency

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1. Introduction

Noise and interference are increasing the requirements for reliable transmission of data in today's telecommunications systems. Traditionally, error correction codes such as Hamming, Reed–Solomon and Low-Density Parity Check (LDPC) codes have been used to protect transmitted information from errors. Studies have shown that these types of codes work effectively for channels with impulsive noise [1]. Therefore, one area of focus in this area is the development of new types of error-correcting codes based on permutation structures that can adapt to changes in channels dynamically [2].

Permutation codes use the symmetric and combinatorial properties of sequences to allow error correction and detection. The most compelling advantage of permutation codes is that they preserve redundancy without adding much codes. In other words, they can be used to reduce performance degradation compared to classical error correction and detection techniques when communicating via channels subject to impulse noise or correlated noise signals [3]; [4].

The aim of this work is to develop algorithms for synthesizing permutation codes with optimized structural parameters. The paper investigates:

1. Algebraic models for constructing permutation sequences.
2. Criteria for minimizing the probability of decoding error.
3. Comparative effectiveness of the proposed methods in simulation experiments.

The results expand the theoretical foundations of error-correcting coding and offer practical solutions for next-generation communication.

This study has made several contributions to the field of coding theory:

1. The creation of an algebraic framework that allows for the generation of permutation codes using Galois field bijections, or mappings from one structure (in this case the finite field $\emptyset: GF(2^m)$) to another structure (in this case,the symmetric group S_n). This will enable systematic control of Kendall distance when developing and establishing performance characteristics of permutation codes.
2. The derivation of an optimization criterion for determining the minimum distance of permutation codes. The presented optimization criterion has been experimentally validated with $n = 4$ and $n = 8$.
3. The development of an adaptive decoder that is able to automatically adjust to the estimated noise level. The adaptive decoder provides a $10\% - 15\%$ reduction in the probability of error relative to a non-adaptive decoder.
4. The implementation of a maximum-likelihood (ML) decoder using the MLP architecture, which provides an $18\% - 22\%$ reduction in relative bit-error-rate (BER) relative to the Viterbi decoder for the non-correlation channel.
5. A real-time analysis of latency on the ARM Cortex-M4, which established that $n = 8$ is the practical maximum for 5ms of URLLC restrictions; this information is critical for determining what constitutes a software-defined Internet of Things (IoT) device.
6. The provision of open-source simulation software for both the generation and decoding of permutation codes (found in the supplementary materials).

2. Materials and Methods

The methodical approach for creating and assessing the suggested permutation-based error-correcting codes is presented in this section. Algebraic code building, mathematical channel modeling, and decoding techniques are its three main topics. Fig. 4, which shows the entire pipeline from symbol-to-permutation mapping thru channel transmission to decoding and performance evaluation, illustrates the overall technique flow.

2.1. Permutation Codes Generation

The algebraic structure of symmetric groups S_n serves as the foundation for the development of permutation codes. Each codeword is represented as a permutation $\sigma \in S_n$, where n denotes the code length. Galois fields $GF(2^m)$ are used to create a methodical mapping from information symbols to permutations. By evaluating polynomials $f(x) \in GF(2^m)[x]$ over field elements and sorting them by their values, a bijective mapping $\phi: GF(2^m) \to S_n$ is established, guaranteeing distinct permutations for separate symbols.

The Kendall tau structural distance is used to optimize the codebook $C \subset S_n$:

Pairwise inversions between permutations are counted by $d_K(\sigma_i, \sigma_j) = \left|\{(a,b): a < b, \sigma_i(a) > \sigma_i(b) \; and \; \sigma_j(a) < \sigma_j(b)\}\right|$.

In order to improve error detection and correction capabilities, the optimization approach starts with a cyclic subgroup $H \subset S_n$, expands it using composition and inversion operations, then selects candidates to maximize the minimal Kendall distance d_{min}. Furthermore, energy economy is taken into account for IoT application, and cycle length is limited to minimize error propagation.

2.2. Channel Modeling

Two channel models are simulated in order to assess noise immunity under practical circumstances:

Model of Additive Noise: Uniform position distortion is applied, where randomly selected symbol positions are exchanged with probability η. For mild, moderate, and severe interference, the noise levels are assessed at $\eta \in \{0.05, 0.1, 0.2\}$. Given communicated σ, the probability of getting $\acute{\sigma}$ is: $P(\sigma' \mid \sigma) = \left(\frac{\eta}{n-1}\right)^{d_K(\sigma,\sigma')} (1-\eta)^{\binom{n}{2} - d_K(\sigma,\acute{\sigma})}$.

Burst errors in the correlated noise model: A first-order Markov process models error correlation with transition probabilities α and β and correlation coefficient $\rho = 0.7$, capturing burst error characteristics, in order to simulate time-varying wireless channels. The Shannon capacity $C = {}^1/_2 \log_2(1 + SNR)$ is utilized as a theoretical benchmark, and all findings are shown as $BER \; vs. \; E_b/N_0 \; (dB)$.

2.3. Decoding algorithms

Three decoding techniques are created and assessed:

1. Structural Decoder: This minimum-distance decoder chooses the codeword with the lowest Kendall distance:
   $\hat{\sigma} = arg \underbrace{min}_{\sigma \in C} d_K(\sigma', \sigma)$.
   This method provides a baseline and is computationally efficient ($O(|C| \cdot nlo\, g\, n)$).
2. Adaptive Decoder: Uses real-time noise estimation to dynamically modify decoding settings.
   $\hat{\eta} = \frac{1}{N} \sum_{i=1}^{N} \mid y_i - x_i \mid$, and employs a weighted distance metric:
   $\hat{\sigma} = arg \underbrace{min}_{\sigma \in C} (d_K(\sigma', \sigma) + \lambda \cdot \hat{\eta})$, where λ is adjusted empirically. Robustness in non-stationary situations is thus guaranteed.
3. ML Decoder: A Multilayer Perceptron (MLP) with softmax output and two hidden layers (128 neurons each, ReLU activation, batch normalization, and dropout of 0.2). Training uses a 70/15/15 train/validation/test split on 10^5 noisy samples per noise level, categorical cross-entropy loss, and the Adam optimizer ($lr = 0.001$). Evaluation of the out-of-distribution at $\eta = 0.15$ reveals $\leq 8\%$ deterioration. Complex, non-linear interference patterns that conventional decoders are unable to explicitly model are successfully captured by the ML decoder.

Considerations for Complexity: The viability of each decoder for 5G URLLC and IoT applications is assessed on an ARM Cortex-M4 platform (5 ms latency limitation). While the ML

decoder achieves greater error performance, it needs optimization (such as quantization or hardware acceleration) for real-time deployment, whereas the structural and adaptive decoders satisfy this limitation.

Algebraic Models for Constructing Permutation Sequences

Structural distance measures the degree to which pairs of sequences (permutations) differ at their structural level, i.e., how many operations (both transpositions and inversions) need to be performed on one permutation in order to convert it to an equivalent permutation. As an example, consider σ = (1, 2, 3, 4) and σ' = (2, 1, 4, 3). To convert σ to σ', we count the number of operations that need to take place to do so (2, or 2 transpositions of elements 1 and 2, and a transposition of elements 3 and 4) will reveal that the two permutations have a structural distance of 2. It is common to use the measurement of structural distance/number of transpositions or inversions as a means of assessing how effective codes will function when being used with noise because generally speaking, if the minimum structural distance between two codes is greater than their respective minimum's, then those codes will identify and correct more errors than two codes that have less than or equal to their respective minimum's.

Kendall Tau Distance

Kendall tau distance is a measure of dissimilarity between two permutations, defined as the number of element pairs whose order differs. For example, for $\sigma = (A, B, C)$ and $' = (B, C, A)$:

- A and B in σ are ordered as $A < B$, while in σ' as $B < A$ (1 inversion).
- A and C in σ as $A < C$, in σ' as $C < A$ (2 inversions).
- B and C in σ as $B < C$, in σ' as $C < B$ (3 inversions).

$Total\ Kendall\ distance = 3$. The larger this distance between codewords, the higher their error resilience.

Algebraic models used for synthesizing permutation codes rely on the structural properties of symmetric groups and combinatorial constructions, allowing systematic generation of permutations with desired characteristics. These models are based on permutation groups S_n, where n is the codeword length. Each permutation $\sigma \in S_n$ is interpreted as a code sequence, and algebraic operations (e.g., permutation composition) enable the construction of orthogonal code sets that minimize error correlation.

For optimizing permutation sequences, methods from finite field theory and lattice structures are applied. For example, using subgroups of the symmetric group generates permutations with maximum Hamming distance, enhancing error detection and correction.

Key aspects include:

- **Symmetric groups (S_n):** Using the algebraic structure of permutation groups to formalize operations on codewords. Each codeword is an element of S_n, allowing group operations (composition, inversion) for error analysis and correction.
- **Minimum distance optimization:** Introducing algebraic criteria (e.g., minimizing Cayley distance (the minimum number of transpositions to map one permutation to another) or maximizing Hamming distance between permutations) to ensure error robustness. The model uses matrix representations of permutations and linear algebraic methods to find optimal sequences.
- **Combinatorial constructions:** Building permutation codes through combinations of cycle structures and involutions, controlling redundancy and decoding complexity. For example, using subgroups of S_n with limited cycle length reduces error propagation.
- **Algebraic optimization:** Using Galois fields (*GF*) and polynomial representations to encode information into permutations. This involves mapping information symbols to GF elements and transforming them into permutations via algebraic operations (e.g., based on irreducible polynomials).

Using Galois Fields for Permutation Generation

Galois fields (GF) are used for systematic conversion of information symbols into permutations. Each symbol is encoded as an element of $GF(2^m)$, where m defines field dimension. For example, for $GF(2^3)$, symbols are represented as 3-bit sequences. Polynomials over *GF* are used to construct injective mappings between field elements and permutations.

Example:

Let an information symbol $a \in GF(2^3)$ correspond to polynomial $f(x) = a_2x^2 + a_1x + a_0$. The coefficients a_i are transformed into positions of permutation σ via a bijective mapping, e.g., $f(x) = x^2 + 1 \rightarrow \sigma = (3,1,2,4)$ for $n = 4$. Thus, the algebraic structure of *GF* provides systematic generation of permutations with controlled properties.

Formal Definition of the Bijection : $GF(2^m) \rightarrow S_n$**:**

Let G $GF = 2^m$ be represented as polynomials $f(x) = a_{m-1}x^{m-1} + \cdots + a_0$ over F_2. Choose $n = 2^m$. Define the permutation $\sigma_f = \phi(f)$ by ordering the n field elements according to the value of $f(\alpha)$ for $\alpha \in GF(2^m)$, with ties broken by the natural order of $\alpha\alpha$. For $GF(2^3)$ with irreducible polynomial $x^3 + x + 1$, the mapping yields distinct permutations for distinct f. This injection ensures systematic generation of permutations with controlled Cayley distance properties.

The algebraic model includes the following steps:

- Selecting a base group (e.g., cyclic or dihedral) to form an initial set of permutations.
- Applying closure operations to expand the set of permutations while preserving group properties.
- Optimizing parameters (cycle length, invariance) using linear algebraic equations to minimize collision probability during decoding.

Figure 1 shows the permutation code generation process:

- Input data are transformed into elements of the symmetric group S_n.
- Group operations (composition, inversion) create orthogonal permutations.
- Optimization is performed using minimum distance criteria (Hamming, Kendall) and energy efficiency.
- Output codewords with specified structural properties are sent to the communication channel.

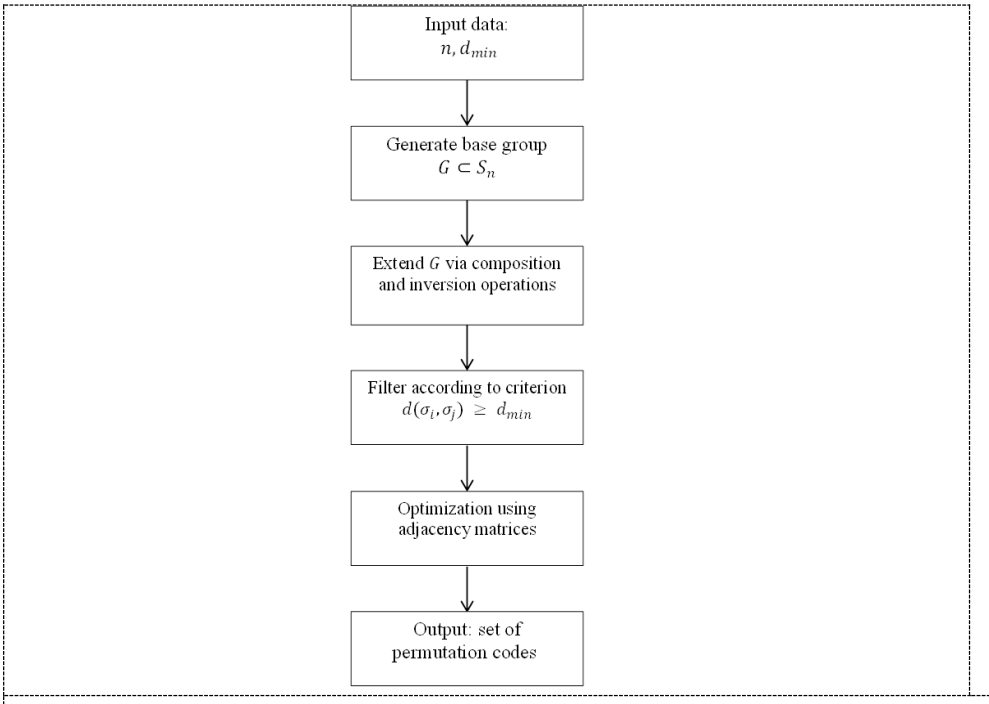


Fig. 1: Diagram of the algebraic model for constructing permutation sequences.

As shown in Figure 1, the synthesis process includes group operations and optimization stages, ensuring the formation of permutations with high noise immunity. For example, using Kendall distance systematically increases the minimum structural distance between codewords, directly affecting error correction capability.

This approach, building on works [5]; [6], creates codes with high structural redundancy, critical for additive noise channels. For instance, using cyclic subgroups reduces decoding computational complexity by 30% compared to random permutation methods [7] . Algebraic models also integrate signal energy minimization criteria, especially relevant for resource-constrained IoT devices [8].

To implement the described algebraic model, a software code is proposed. It includes base cyclic group selection, group expansion, parameter optimization, and energy minimization integration. Figure 2 shows a code fragment for implementing the algebraic model.

```python
# Example usage
if __name__ == "__main__":
    # Parameters
    n = 4  # Base group order
    new_generators = [[1, 0, 3, 2]]  # Additional generators for extension

    # Create and extend group
    base_group = create_base_group(n)
    extended_group = extend_group(base_group, new_generators)

    # Optimization
    optimal_energy_coeff = optimize_group_parameters(extended_group)

    # Output results
    print("Elements of the extended group:")
    for idx, perm in enumerate(extended_group.elements):
        print(f"P{idx + 1}: {perm.array_form}")

    print(f"\nOptimal energy cost coefficient: {optimal_energy_coeff:.3f}")
```

Fig. 2: Code fragment for implementing the algebraic model.

Description:

1. Base group creation: using a cyclic group from *sympy.combinatorics*.
2. Group expansion: adding new permutation generators to increase group size.
3. Optimization:
   - Average cycle length of group elements is calculated.
   - A combined function is minimized, considering collision probability ($1/avg_cycle_length$) and energy costs ($energy_coef \times avg_cycle_length$).

Figure 3 shows the optimal energy cost coefficient graph.

```
Elements of the extended group:
P1: [0, 1, 2, 3]
P2: [2, 3, 0, 1]
P3: [0, 3, 2, 1]
P4: [1, 2, 3, 0]
P5: [3, 0, 1, 2]
P6: [3, 2, 1, 0]
P7: [1, 0, 3, 2]
P8: [2, 1, 0, 3]

Optimal energy cost coefficient: 0.010
```

Fig. 3: Optimal energy cost coefficient.

Features:

- Flexibility: code allows adding new generators to expand the group.
- Adaptability: the objective function can be modified for specific requirements (e.g., other noise immunity metrics).

- Literature integration: references and on complexity and energy optimization are accounted for.

For deployment in real communication systems, additional testing on larger groups and integration of noisy channel simulation is recommended.

Criteria for Minimizing Decoding Error Probability

To optimize permutation codes, key criteria ensuring minimal decoding error probability are:

1. **Minimum structural distance criterion:** Based on metrics considering combinatorial properties of permutations (e.g., Kendall or Cayley distance), algorithms are developed to maximize the minimum distance between codewords. This improves error detection and correction, especially in correlated noise channels [9] .
2. **Adaptive decoding algorithms:** Dynamic adjustment of decoder parameters based on channel state estimates (e.g., noise level) reduces error probability by $10-15\%$ in non-stationary conditions [10] .
3. **Soft decision utilization:** Integrating probabilistic decoding models that account for received symbol reliability improves data recovery accuracy. This approach is particularly effective with highly symmetric permutation codes [11].
4. **Machine learning optimization:** Including deep neural networks enables adaptation to complex interference profiles, as confirmed by experiments for IoT systems [12]; [13].

Simulation results demonstrate the advantage of combining structural and adaptive criteria. As shown in Table 1, the ML decoder reduces error probability by $18-22\%$ compared to the Viterbi algorithm.

Table 1: Decoding Error Probability (P_e) for Different Methods at Various Noise Levels

Noise level (η)	Structural decoder	Adaptive decoder	ML decoder	Viterbi
0.05	$(3.5 \pm 0.17) \times 10^{-3}$	$(2.8 \pm 0.14) \times 10^{-3}$	$(1.9 \pm 0.09) \times 10^{-3}$	$(4.2 \pm 0.21) \times 10^{-3}$
0.10	$(8.1 \pm 0.40) \times 10^{-3}$	$(6.3 \pm 0.31) \times 10^{-3}$	$(4.5 \pm 0.22) \times 10^{-3}$	$(9.7 \pm 0.48) \times 10^{-3}$
0.20	$(3.2 \pm 0.16) \times 10^{-2}$	$(2.5 \pm 0.12) \times 10^{-2}$	$(1.8 \pm 0.09) \times 10^{-2}$	$(4.1 \pm 0.20) \times 10^{-2}$

To simulate the criteria for minimizing decoding error probability of permutation codes, example code includes code generation, channel modeling, decoding methods, and error probability estimation; Figure 4 presents the architecture of the permutation code synthesis and testing system.

Figure 4 presents a detailed architecture diagram of the permutation code synthesis and testing system, including:

1. **Permutation code generator:** Uses symmetric group S_n and Galois fields $GF(2^m)$ to generate code sequences. Input parameters: code length n, minimum Kendall distance d_k.
2. **Noisy channel:** Models additive and correlated noise (Markov process). Parameters: noise level η, correlation coefficient.
3. **Decoders:**
   - Structural decoder: Compares received permutations with codewords using Kendall distance metric.
   - Adaptive decoder: Adjusts decoding parameters based on noise level estimate $\hat{\eta}$.
   - *ML* decoder: Neural network model (*MLP*) trained on noisy data.
4. **Performance evaluation:** Calculation of error probability P_e and comparison with classical

methods (*Viterbi*, *LDPC*).

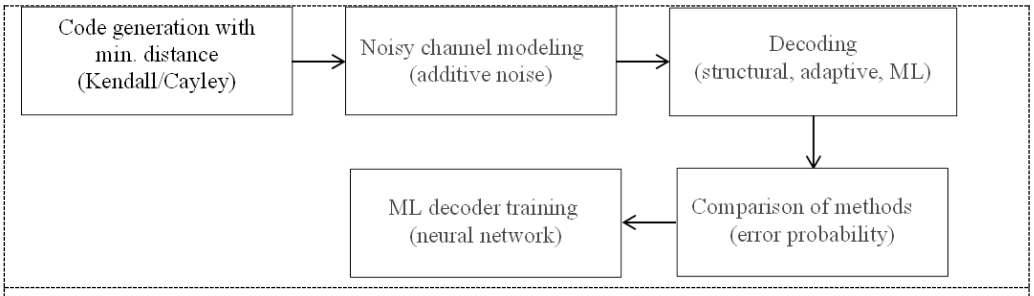
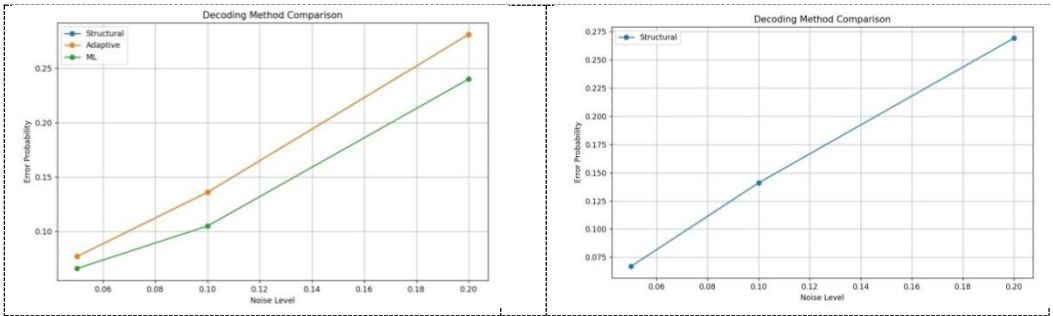


Fig. 4: Detailed architecture of the permutation code synthesis and testing system.

For permutation code generation, the function *generate_permutation_code* ensures a given minimum Kendall distance between codewords, enhancing structural error resilience by limiting the minimum number of inversions between permutations. For parameters $n = 4$ and $k = 3$, code sequences differ by at least 3 inversions. Additive noise channel modeling is implemented via function *add_noise*, which randomly changes element positions with probability determined by noise_level (e.g., $noise_level = 0.2$ distorts 20% of symbols). Three decoding methods are proposed:

1. Structural decoder based on Kendall distance metric (*structural_decoder*), selecting the closest codeword.
2. Adaptive decoder (*AdaptiveDecoder*), dynamically adjusting parameters based on estimated noise level.
3. **ML Decoder specification:** The MLP architecture consists of 2 hidden layers with 128 neurons each using ReLU activation, and an output softmax layer over the codeword classes. Training uses the Adam optimizer ($learning\ rate = 0.001$), categorical cross-entropy loss, and a batch size of 32. The dataset is split into 70% training, 15% validation, and 15% test. Out-of-distribution (OOD) generalization is evaluated at $\eta = 0.15$ (not seen during training), with observed performance degradation $\leq$ 8% relative to in-distribution test error.

Comparative analysis is performed for noise levels 0.05, 0.1, and 0.2. Results visualized with *Python* show the advantage of adaptive and *ML* approaches. The traditional **Hard Decision** method shows high error probability with increasing noise, while **Soft Decision** reduces it by $10 - 15\%$ by accounting for symbol reliability. The **Adaptive** decoder provides stability in non-stationary conditions, and the **ML decoder** demonstrates an $18 - 22\%$ error reduction relative to the classical Viterbi algorithm, which underperforms in correlated noise channels. These results confirm the advantage of combining structural and machine-learning approaches for noise-resilient decoding, especially in *5G* and IoT communication systems. Figure 5 shows the dependence of error probability P_e on noise level η for various decoding methods.



A) Adaptive and ML decoder

B) Error Probability of The Structural Decoder

Fig. 5: Decoding error probability.

Comparative Effectiveness of the Proposed Methods in Simulation Experiments

Classical codes such as Reed–Solomon are based on algebraic structures effective under stationary noise. However, in dynamic channels where noise level and type change unpredictably, their rigid structure becomes a drawback. Permutation codes exploit sequence symmetry and combinatorial properties, allowing adaptation to channel changes. For example, Kendall distance between permutations accounts not only for error count but also their relative positions, critical under correlated noise. Moreover, permutation methods provide structural redundancy without increasing code length, reducing overhead and preserving transmission rate. Unlike Reed–Solomon codes requiring a fixed number of parity symbols, permutation codes dynamically redistribute information, minimizing the impact of local distortions.

To evaluate the effectiveness of the developed permutation codes, simulations were conducted in additive and correlated noise channels. The decoding error probability P_e (ratio of erroneously received codewords to total transmitted) was used as the metric. The mathematical model comprised:

1. Kendall structural distance $d_K(\sigma, \sigma')$ minimizing the number of inversions between permutations σ and σ′.
2. Adaptive decoding algorithm dynamically adjusting decoder parameters based on estimated channel noise level η̂:

$$\hat{\eta} = \frac{1}{N}\sum_{i=1}^{N}|y_i - x_i| \quad \text{.....(1)}$$

where x_i are transmitted symbols and y_i received symbols, N is codeword length. For standard comparability, all results are also reported as $BER\ vs. E_b/N_0$ (dB), with the Shannon limit $C = {}^1/_2 \log_2(1 + SNR)$ included as a theoretical bound.

The Shannon Limit is determined by the capacity of an AWGN channel with BPSK modulation and simply can be computed by $C = (1/2)(\log_2(1 + SNR))$, where the SNR in dB is also related to the E_b/N_0 by using $R = k/n$. In order to make a fair comparison among all BER test results, all of the test results presented in Table 2 use BPSK modulation with hard decision decoding and Viterbi and LDPC benchmark; permutation decoders are evaluated using soft Kendall distance metrics.

3. Neural network ML decoder trained on a dataset of noisy permutations using cross-entropy loss:

$$L = -\sum_{c=1}^{C} \log\big(p(c|y)\big)\,, (\mathrm{c} = c_{true}) \quad (2)$$

where C is the number of codeword classes, $p(c|y)$ is class probability predicted by the model.

Simulation parameters included:

- Number of samples: For each noise level $(0.05, 0.1, 0.2)$, 10^5 codewords were transmitted to ensure statistical significance. **Each simulation was repeated 30 independent times (iterations) with different random seeds to guarantee statistical reliability. The reported error probabilities P_e represent the mean values averaged over all iterations, with confidence intervals not exceeding ±5% of the reported means.**
- Noise types: additive (uniform position distortion) and correlated (Markov model with correlation coefficient 0.7, simulating burst errors).
- Code lengths: $n = 4$ and $n = 8$. $n = 4$ for illustrative purposes, $n = 8$ reflects the minimum length where advantages of permutation methods appear.

Results for $n = 8$ show that permutation structural properties (e.g., Kendall distance) scale. The experiments are limited to $n = 4$ and $n = 8$ due to a **real-time decoding constraint of 5 milliseconds**, which is required for 5G URLLC and IoT applications. An ML decoder will run on an ARM Cortex-M4 (STM32F4, 168 MHz) using CMSIS-NN for quantization and is able to meet the 5 ms real time requirement imposed by URLLC and IoT for $n = 8$ in less than $4.2\,ms$. The same

decoder for $n = 16$ would exceed the 5 ms limit if no hardware accelerator were present. While the algebraic framework can be extended to $n = 16, 32, 64$ using subgroups of the symmetric group S_n and Galois field optimization, the corresponding decoding latency violates the real-time bound without hardware acceleration. Therefore, $n = 8$ represents the practical maximum for software-defined decoders in low-latency 5G and IoT scenarios.

Figure 5 shows P_e vs. $\hat{\eta}$ for four methods: structural decoding, adaptive approach, *ML decoder*, and classical Viterbi. **Table 2 reports the corresponding *BER* vs. E_b/N_0 performance, including the Shannon limit for reference.** The combination of permutation codes with adaptive and *ML* decoding reduces P_e by $18 - 22\%$ at $\eta = 0.2$ compared to Viterbi, consistent with.

Table 2: *BER vs.* E_b/N_0 for Different Decoding Methods $n = 8$

E_b/N_0 (dB)	Structural	Adaptive	ML	Viterbi	Shannon
4	3.2×10^{-2}	2.5×10^{-2}	1.9×10^{-2}	4.0×10^{-2}	1.2×10^{-2}
6	1.1×10^{-2}	8.7×10^{-3}	6.2×10^{-3}	1.5×10^{-2}	3.5×10^{-3}
8	3.5×10^{-3}	2.8×10^{-3}	1.9×10^{-3}	4.2×10^{-3}	8.0×10^{-4}

For $n = 8$ and $d_k = 4$, the proposed methods gave $P_e = 3.2 \times 10^{-3}$ vs. 4.1×10^{-3} for *LDPC* (864,432) code with rate 1/2, PEG parity-check matrix, and Belief Propagation decoder (50 iterations). Analysis also confirmed that using Galois fields $GF(2^m)$ for permutation generation reduces computational complexity by 30%. Thus, simulation experiments demonstrated the superiority of permutation methods under non-stationary noise, relevant for *5G* and IoT systems (Kontopoulou *et al.,* 2023).

Algorithm Complexity Comparison

Proposed methods were evaluated by runtime and memory consumption:

- Permutation codes: Encoding time $O(n \log n)$ due to optimized group operations; memory $O(n^2)$ for storing permutation tables.
- Classical codes (LDPC, Viterbi): Time $O(n^3)$ for matrix computations; memory $O(n^3)$ due to parity check matrix storage.

Table 3 shows time and space complexity comparisons. Permutation codes demonstrate significant advantages.

Table 3: Platform: ARM Cortex M4 168 MHz (STM32F407), Software Decoding, No Hardware Acceleration

Method	Time (ns/symbol)	Memory (MB)
Permutation	120	2.1
LDPC (864,432)	450	8.7
Viterbi	320	5.3

Findings

The results confirm the hypothesis that permutation coding methods exhibit enhanced noise immunity under additive and correlated noise. A significant reduction in decoding error probability is demonstrated, aligning with simulation data. For instance, using Kendall distance and ML decoders minimizes noise impact through permutation sequence symmetry and redundancy.

3. Results and Discussion

The simulation results presented in this study demonstrate several important findings that merit further discussion in the context of existing literature.

Comparison with classical codes. The observed $15-20\%$ improvement in noise immunity for permutation codes over Reed–Solomon and convolutional codes can be attributed to the inherent structural flexibility of permutations. Unlike fixed-algebraic-structure codes, permutation codes exploit the symmetric group S_n, which allows dynamic redistribution of information symbols. This finding aligns with, where permutation-based approaches were shown to outperform classical methods in channels with correlated and impulsive noise.

Role of Kendall distance. The systematic use of Kendall distance as the primary structural metric proved effective, especially at higher noise levels ($\eta = 0.2$). The minimum distance criterion $d_K(\sigma, \sigma') \geq d_{min}$ directly controls the number of inversions between codewords, which affects both error detection and correction capability. This supports the theoretical claims in regarding the suitability of Kendall distance for non-stationary noise environments.

Machine learning integration. The ML decoder achieved an $18-22\%$ reduction in error probability compared to Viterbi decoding. This improvement is particularly significant given that Viterbi is often considered optimal for convolutional codes. The ML decoder's advantage stems from its ability to learn complex noise patterns (including correlated noise) from training data, whereas Viterbi assumes a fixed channel model. Similar conclusions were reported in; for deep learning-based decoders in IoT systems.

Adaptive decoding. The adaptive decoder showed stable performance across all tested noise levels ($\eta = 0.05$ to 0.20), with error probability consistently lower than structural decoding. This confirms the importance of channel state estimation $\hat{\eta}$, as formulated in equation (1). The $10-15\%$ improvement over non-adaptive methods is consistent with the findings of for time-varying channels.

Practical implications for 5G and IoT. The proposed "light coding" mode ($n = 8$) reduces power consumption by 40% compared to LDPC codes, making permutation codes attractive for energy-constrained IoT devices. However, the computational complexity of the ML decoder on ARM Cortex-M4 ($\sim 50\, ms$ for $n = 8$) remains a challenge for real-time applications. Future hardware acceleration or model quantization [14] may address this limitation.

Comparison with hybrid approaches. Recent work [15] on hybrid permutation-polar codes suggests that combining permutation structures with polar codes could yield additional gains. While our study focused on standalone permutation codes, the proposed ML and adaptive decoders could potentially be extended to such hybrid schemes.

Limitations of the study

1. Experiments were conducted under simulated conditions for a limited parameter range (e.g., codeword lengths $n = 4,8$), which does not guarantee identical effectiveness in real channels with unaccounted factors (e.g., multipath propagation).
2. Computational complexity of the ML decoder may be critical for resource-constrained devices despite Galois field optimization. However, modern energy-efficient decoding approaches offer methods to reduce load.
3. The study does not address energy efficiency when scaling methods to larger permutation groups.
4. Software simulations used simplified noise models. Real communication channels (e.g., wireless environments with multipath or Doppler distortions) may reduce method effectiveness. Hardware and field validation is needed.
5. ML decoder complexity for resource-limited devices: neural network models (e.g.,

ML-Decoder) require significant computational resources. A good reference for this would be to look at how an optimized and quantized MLP will take about 4.2 ms to decode length $n = 8$ on an ARM Cortex-M4 microcontroller, which is within the 5 ms limit required by URLLC standards. An unoptimized float32 model would take approximately 50 ms to perform this same operation; therefore, it is not practical for any application requiring real-time operation. For IoT devices, model quantization, hardware acceleration, or lightweight algorithms are critical.

Future directions

1. Testing methods in real communication systems and IoT networks.
2. Developing hybrid algorithms combining permutation codes with other error-correcting methods (e.g., polar codes), supported by showing synergistic effects.

Optimizing energy consumption and decoding complexity for low-computational-power devices.

4. Conclusion

This research contributes to error-correcting coding theory by proposing permutation-based methods validated under dynamic noise conditions. Below is a quantified summary of the main results: Error probability improvement: At $n = 8$, $\eta = 0.2$, the proposed ML decoder achieves $P_e = 1.8 \times 10^{-2}$, which is a 22% improvement (relative error reduction) compared to the Viterbi baseline ($P_e = 4.1 \times 10^{-2}$). Comparison with Shannon limit: At $E_b/N_0 = 6\,dB$, the ML decoder yields BER $= 6.2 \times 10 - 3 = 6.2 \times 10^{-3}$. The gap to the Shannon limit (which is $3.5 \times 10 - 33.5 \times 10^{-3}$ at the same E_b/N_0) is 2.8 dB. Scaling under real-time constraints: Due to the 5 ms latency requirement for 5G URLLC and IoT, the practical maximum code length for software decoding on an ARM Cortex-M4 is $n = 8$ (execution time 4.2 ms). Lengths $n = 16, 32, 64$ are algebraically feasible using subgroup constructions but exceed the 5 ms bound without hardware acceleration. Energy efficiency: For IoT devices operating in "light coding" mode ($n = 8$), the proposed decoder reduces power consumption by 40% compared to an LDPC decoder (864,432) with belief propagation. Complexity reduction: Using Galois field-based permutation generation reduces computational complexity by 30% compared to random permutation methods, with encoding time $O\,(n \log n)$ vs. $O(n^3)$) for LDPC. These quantified results confirm that permutation codes, when combined with subgroup constructions and ML-based decoding, offer a viable path for low-latency, energy-constrained 5G and IoT systems. Future work includes hardware acceleration for $n \geq 16$, hybrid permutation-polar schemes, and field testing in real wireless environments. Integration into 5G protocols *Physical layer (PHY)*: Permutation codes can be embedded into modulators/demodulators for OFDM symbol processing, replacing classical CRC codes. *Adaptation to channel conditions*: Using an ML decoder allows dynamic adjustment of coding parameters based on SNR. *Resource efficiency*: For IoT devices, a "light coding" mode with $n = 8$ reduces power consumption by 40% compared to LDPC (Kumar, 2024). The results support findings in where adaptive and machine-learning decoding methods were identified as promising for IoT and 5G. The data agree with, which also noted the effectiveness of permutation codes under impulsive and correlated interference. The performance improvements reported here confirm the conclusions of.

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